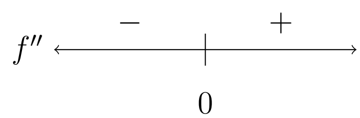
1. Find each value of x where $f(x) = e^x - 100x - \frac{1}{2}x^2$ has an inflection point.

Solution. An inflection point of f is a point in the domain of f where f'' changes sign. Note that the domain of f is all real numbers, and thus we may proceed with finding the second derivative of f.

$$\begin{aligned} f'(x) &= e^x - 100 - x \implies \\ f''(x) &= e^x - 1. \end{aligned}$$

Our only potential inflection point occurs where $f''(x) = 0$, meaning $x = 0$. We exhibit the sign line for f'' below.



Indeed, f'' changes sign at $x = 0$, so it is our only inflection point.

$$\boxed{x = 0}$$

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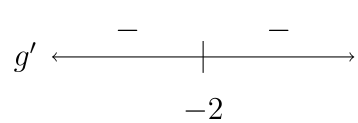
2. Let $g(x) = \frac{1}{2x+4}$. Find each interval where g is decreasing and concave up.

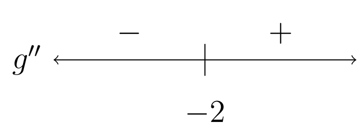
Solution. Note that the domain of g is $(-\infty, -2) \cup (-2, \infty)$. Now, our function g is decreasing where its first derivative is negative, and g is concave up where its second derivative is positive, so we construct sign lines for both g' and g''. First, we compute the first and second derivatives of g.

$$g'(x) = \frac{-2}{(2x+4)^2} \qquad g''(x) = \frac{8}{(2x+4)^3}$$

The points which should be marked on our g' sign line are those points where g' is either 0 or undefined. Since g' never takes on the value 0, we just note that it is undefined at $x = -2$, so that goes on our number line. Then, we check points in each interval on our number line to determine where g' is positive or negative.



The procedure for the the g'' sign line is the same, and the sign line is given below.



Now, g is both decreasing and concave up on $(-2, \infty)$, so this is our answer.

$$\boxed{(-2, \infty)}$$

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3. Evaluate

$$\lim_{x \to \infty} \frac{\ln(x)}{x^2}.$$

Solution. Since 'plugging in' infinity gives an indeterminate form, we apply L'Hospital's Rule.

$$\begin{aligned} \lim_{x \to \infty} \frac{\ln(x)}{x^2}. &= \lim_{x \to \infty} \frac{\frac{d}{dx}(\ln(x))}{\frac{d}{dx}(x^2)} \\ &= \lim_{x \to \infty} \frac{\frac{1}{x}}{2x} \\ &= \lim_{x \to \infty} \frac{1}{2x^2} \\ &= 0. \end{aligned}$$

$$\boxed{0}$$