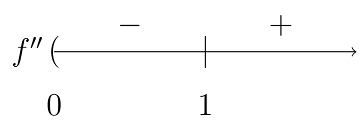
1. Find each value of x where $f(x) = \ln(x) + 100x + \frac{1}{2}x^2$ has an inflection point.

   Solution. An inflection point of f is a point in the domain of f where f'' changes sign. First, note that the domain of f is $(0, \infty)$, Now, we proceed with finding the second derivative of f.

$$\begin{aligned} f'(x) &= \frac{1}{x} + 100 + x \implies \\ f''(x) &= \frac{-1}{x^2} + 1 \\ &= \frac{x^2 - 1}{x^2}. \end{aligned}$$

   Our potential inflection points occur where either f'' is 0 or undefined. Since f'' is only undefined at the point $x = 0$, which is not in the domain of f, we concern ourselves only with those points where f'' is 0. If $f''(x) = 0$, then $x^2 = 1$. Thus $f''(x) = 0$ means $x = -1$ or $x = 1$. Since $x = -1$ is not in our domain, our only potential inflection point is $x = 1$.



   Indeed, f'' changes sign at $x = 1$, so it is our only inflection point.

$$\boxed{x = 0}$$

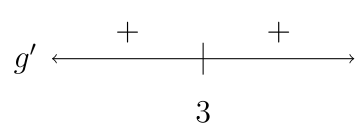
---

2. Let $g(x) = \frac{1}{3-x}$. Find each interval where g is increasing and concave up.

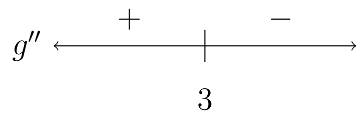
   Solution. Note that the domain of g is $(-\infty, 3) \cup (3, \infty)$. Now, our function g is increasing where its first derivative is positive, and g is concave up where its second derivative is positive, so we construct sign lines for both g' and g''. First, we compute the first and second derivatives of g.

$$g'(x) = \frac{1}{(3-x)^2} \qquad g''(x) = \frac{2}{(3-x)^3}$$

   The points which should be marked on our g' sign line are those points where g' is either 0 or undefined. Since g' never takes on the value 0, we just note that it is undefined at $x = 3$, so that goes on our number line. Then, we check points in each interval on our number line to determine where g' is positive or negative.



The procedure for the the g'' sign line is the same, and the sign line is given below.



Now, g is both increasing and concave up on $(-\infty, 3)$, so this is our answer.

$$\boxed{(-\infty, 3)}$$

---

3. Evaluate
$$\lim_{x\to\infty} \frac{e^x}{x^2}.$$

Solution. Since 'plugging in' infinity gives an indeterminate form, we apply L'Hospital's Rule.

$$\begin{aligned} \lim_{x\to\infty} \frac{e^x}{x^2} &= \lim_{x\to\infty} \frac{\frac{d}{dx}(e^x)}{\frac{d}{dx}(x^2)} \\ &= \lim_{x\to\infty} \frac{e^x}{2x} \end{aligned}$$

Again, 'plugging in' infinity gives an indeterminate form, so we apply L'Hospital's Rule again.

$$\begin{aligned} \lim_{x\to\infty} \frac{e^x}{2x} &= \lim_{x\to\infty} \frac{\frac{d}{dx}(e^x)}{\frac{d}{dx}(2x)} \\ &= \lim_{x\to\infty} \frac{e^x}{2} \\ &= \infty. \end{aligned}$$

$$\boxed{\infty}$$