Name ______________

Quiz 7

1. Find the equation of the tangent line of $f(x) = \ln(\sec(x))$ at $x = 0$.

   First, we find compute $f'(0)$:

$$\begin{aligned} f'(x) &= \frac{\mathrm{d}}{\mathrm{dx}} \ln(\sec(x)) = \frac{\mathrm{d}}{\mathrm{dx}} \ln(\cos^{-1}(x)) \\ &= \frac{\mathrm{d}}{\mathrm{dx}} \Big(-\ln(\cos(x)) \Big) \\ &= -\frac{\frac{\mathrm{d}}{\mathrm{dx}} \cos(x)}{\cos(x)} \\ &= -\frac{-\sin(x)}{\cos(x)} = \tan(x). \end{aligned}$$

   So

$$f'(0) = \tan(0) = 0.$$

   Our tangent line should pass through $(0, f(0)) = (0, 0)$, so our line is $y - 0 = 0(x - 0)$, or more simply

$$y = 0.$$

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2. Find the derivatives of the given functions:

   (a) $f(x) = \log_3(x)$

$$\begin{aligned} f'(x) &= \frac{\mathrm{d}}{\mathrm{dx}} \left(\frac{\ln(x)}{\ln(3)} \right) \\ &= \frac{1}{\ln(3) \cdot x} \end{aligned}$$

(b) $f(x) = \arccos(x^3)$

$$\begin{aligned} f'(x) &= \frac{\mathrm{d}}{\mathrm{dx}} \arccos(x^3) \\ &= \frac{-1}{\sqrt{1-(x^3)^2}} \cdot \frac{\mathrm{d}}{\mathrm{dx}}(x^3) \\ &= \frac{-3x^2}{\sqrt{1-x^6}} \end{aligned}$$

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3. Find the derivative of $f(x) = x^{(e^x)}$.

   Let $y = x^{(e^x)}$. We find $f'(x) = y'$ using logarithmic differentiation:

$$\begin{aligned} \ln(y) &= \ln(x^{(e^x)}) = e^x \cdot \ln(x) \implies \\ \frac{\mathrm{d}}{\mathrm{dx}} \ln(y) &= \frac{\mathrm{d}}{\mathrm{dx}} \Big(e^x \cdot \ln(x)\Big) \\ \frac{y'}{y} &= e^x \frac{\mathrm{d}}{\mathrm{dx}} \ln(x) + \ln(x) \frac{\mathrm{d}}{\mathrm{dx}} e^x \\ &= e^x \cdot x^{-1} + e^x \cdot \ln(x) \implies \\ y' &= e^x(x^{-1} + \ln(x)) \cdot y \\ &= e^x(x^{-1} + \ln(x)) \cdot x^{(e^x)} \\ &= e^x \cdot x^{(e^x)-1} + e^x \cdot \ln(x) \cdot x^{(e^x)} \end{aligned}$$

Name ______________

Quiz 7

1. Find the equation of the tangent line of $f(x) = \ln(\csc(x))$ at $x = \frac{\pi}{2}$.

   First, we find compute $f'(\frac{\pi}{2})$:

$$\begin{aligned} f'(x) &= \frac{\mathrm{d}}{\mathrm{dx}} \ln(\csc(x)) = \frac{\mathrm{d}}{\mathrm{dx}} \ln(\sin^{-1}(x)) \\ &= \frac{\mathrm{d}}{\mathrm{dx}} \Big(-\ln(\sin(x)) \Big) \\ &= -\frac{\frac{\mathrm{d}}{\mathrm{dx}} \sin(x)}{\sin(x)} \\ &= -\frac{\cos(x)}{\sin(x)} = -\cot(x). \end{aligned}$$

   So

$$f'(\frac{\pi}{2}) = -\cot(\frac{\pi}{2}) = 0.$$

   Our tangent line should pass through $(\frac{\pi}{2}, f(\frac{\pi}{2})) = (\frac{\pi}{2}, 0)$, so our line is $y - 0 = 0(x - \frac{\pi}{2})$, or more simply

$$y = 0.$$

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2. Find the derivatives of the given functions:

   (a) $f(x) = 3^x$

$$\begin{aligned} f'(x) &= \frac{\mathrm{d}}{\mathrm{dx}}(3^x) \\ &= \ln(3) \cdot 3^x \end{aligned}$$

(b) $f(x) = \arcsin(x^2)$

$$\begin{aligned} f'(x) &= \frac{\mathrm{d}}{\mathrm{dx}} \arcsin(x^2) \\ &= \frac{1}{\sqrt{1-(x^2)^2}} \cdot \frac{\mathrm{d}}{\mathrm{dx}}(x^2) \\ &= \frac{2x}{\sqrt{1-x^4}} \end{aligned}$$

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3. Find the derivative of $f(x) = x^{\ln(x)}$.

   Let $y = x^{\ln(x)}$. We find $f'(x) = y'$ using logarithmic differentiation:

$$\begin{aligned} \ln(y) &= \ln(x^{\ln(x)}) = \ln(x) \cdot \ln(x) \\ &= (\ln(x))^2 \implies \\ \frac{\mathrm{d}}{\mathrm{dx}} \ln(y) &= \frac{\mathrm{d}}{\mathrm{dx}} (\ln(x))^2 \\ \frac{y'}{y} &= 2 \cdot \ln(x) \cdot \frac{\mathrm{d}}{\mathrm{dx}} \ln(x) \\ &= 2\ln(x) \cdot x^{-1} \implies \\ y' &= 2\ln(x) \cdot x^{-1} \cdot y \\ &= 2\ln(x) \cdot x^{-1} \cdot x^{\ln(x)} \\ &= 2\ln(x) \cdot x^{\ln(x)-1} \end{aligned}$$