1. Find an expression for the exact area under $f(x) = x^3 + 1$ from $x = 0$ to $x = 3$ as the limit of a Riemann sum with n subintervals of equal width. DO NOT EVALUATE IT!

   Solution. For convenience, we use the right Riemann sum with n subintervals. The formula for rectangle width, Δx, and the formula for where we take our i^{th} rectangle height, x_i, are straight from the definitions,

$$\begin{aligned} \Delta x = & \frac{3-0}{n} & = \frac{3}{n} \text{ and} \\ x_i = & \ 0 + i\Delta x & = \frac{3i}{n}. \end{aligned}$$

   Now, the formula for the exact area under $f(x)x^3 + 1$ from $x = 0$ to $x = 3$ as the limit of a Riemann sum is again from definition,

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta_x = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(\frac{3i}{n} \right)^3 + 1 \right) \frac{3}{n}.$$

$$\boxed{\lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(\frac{3i}{n} \right)^3 + 1 \right) \frac{3}{n}}$$

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2. Find $g(x)$, given $g'(x) = \frac{1}{x} + x^3 + e^{2x}$ and $g(1) = \frac{1}{4}$.

   Solution. We carefully antidifferentiate $g'(x)$ and deduce the constant C using $g(1) = \frac{1}{4}$. Using $\int \frac{1}{x}dx = \ln|x|$ with $\int x^3 dx = \frac{1}{4}x^4$ and $\int e^{2x}dx = \frac{1}{2}e^{2x}$, we have

$$g(x) = \ln|x| + \frac{1}{4}x^4 + \frac{1}{2}e^{2x} + C.$$

   Asserting

$$g(1) = \ln(1) + \frac{1}{4}(1)^4 + \frac{1}{2}e^2 + C = 0 + \frac{1}{4} + \frac{e^2}{2} + C = \frac{1}{4},$$

   we have that $C = -\frac{e^2}{2}$, meaning $g(x) = \ln|x| + \frac{1}{4}x^4 + \frac{1}{2}e^{2x} - \frac{e^2}{2}$.

$$\boxed{g(x) = \ln|x| + \tfrac{1}{4}x^4 + \tfrac{1}{2}e^{2x} - \tfrac{e^2}{2}}$$