1. Find an expression for the exact area under $f(x) = (x-1)^2$ from $x = 1$ to $x = 3$ as the limit of a Riemann sum with n subintervals of equal width. DO NOT EVALUATE IT!

   Solution. For convenience, we use the right Riemann sum with n subintervals. The formula for rectangle width, Δx, and the formula for where we take our i^{th} rectangle height, x_i, are straight from the definitions,

$$\begin{aligned} \Delta x = & \frac{3-1}{n} & = \frac{2}{n} \text{ and} \\ x_i = & 1 + i\Delta x & = 1 + \frac{2i}{n}. \end{aligned}$$

   Now, the formula for the exact area under $f(x) = (x-1)^2$ from $x = 0$ to $x = 3$ as the limit of a Riemann sum is again from definition,

$$\begin{aligned} \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta_x &= \lim_{n\to\infty} \sum_{i=1}^{n} \left(1 + \frac{2i}{n} - 1\right)^2 \frac{2}{n} \\ &= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2i}{n}\right)^2 \frac{2}{n} \\ &= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{8i^2}{n^3} \end{aligned}$$

$$\boxed{\lim_{n\to\infty} \sum_{i=1}^{n} \frac{8i^2}{n^3}}$$

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2. Find $g(x)$, given $g'(x) = \frac{1}{x^2} + \frac{1}{1+x^2} + e^x$ and $g(1) = e$.

   Solution. We carefully antidifferentiate $g'(x)$ and deduce the constant C using $g(1) = e$. Using $\int \frac{1}{x^2} dx = -\frac{1}{x}$ with $\int \frac{1}{1+x^2} dx = \arctan(x)$ and $\int e^x dx = e^x$, we have

$$g(x) = -\frac{1}{x} + \arctan(x) + e^x + C.$$

   Asserting

$$g(1) = -1 + \frac{\pi}{4} + e + C = e,$$

   we have that $C = 1 - \pi/4$, meaning $g(x) = -\frac{1}{x} + \arctan(x) + e^x + 1 - \pi/4$.

$$\boxed{g(x) = -\tfrac{1}{x} + \arctan(x) + e^x + 1 - \pi/4}$$