FALL 2018 – MAS 4301 — ABSTRACT ALGEBRA — HOMEWORK 1

NAME: ___________________________________

INSTRUCTIONS:

• Due Friday, August 31, 2018 either at the beginning of class or by 4pm at LIT 408.

• Staple this cover-sheet (or reproduction) to your homework.

• **Solutions versus answers:** Solutions, rather than answers, are expected for all problems. Even for non-proof problems, an answer alone (such as "23", "yes", etc) is not sufficient; you need to show how you arrived at the answer.

• **About proofs:** Many of the problems require formal proofs. Proofs must be properly written up, with correct mathematical notation and terminology, and in complete sentences. Use the examples from class and the textbook as models for your own proofs. In your proofs, you can use any result covered in class, the notes or the textbook. When using such a result, say so and cite the specific result used (e.g., "by Theorem 6.2(1) in Gallian" or "by a result from class in Chapter 6 ").

• Solutions to be problems should be written in a proper and coherent manner. Write in complete sentences. Explain your reasoning. All work should be handwritten and neat. Write in such a way that any student in the class can follow your work. Use examples from class and the textbook as models for your work.

• **Getting help:** You may use the textbook, class notes and get help from other class members as well as Dr.G. However help is restricted to discussion only. Your final solution must be written up independently and in your own words. It is not permitted to copy someone else's work or copy solutions from anywhere. Any help on any question must explicitly cited giving details. See examples below.

DETAILED ACKNOWLEDGMENT OF OUTSIDE HELP: (Included details here or in the body of your solution)

EXAMPLES OF DETAILED ACKNOWLEDGEMENT OF OUTSIDE HELP:

(1) Me and Mary K. discussed Problem 1. We found a solution together by guessing. On my own I found a difference solution. But were stuck on finding infinitely many solutions. Then we saw how to use Dr.G's hint that $11 \cdot 13 - 13 \cdot 11 = 0$. After our discussion and without using notes we wrote up our solutions independently.

(2) I asked Dr.G. for help with Problem 2. Dr.G. suggested letting $r = a \pmod m = b \pmod m$ and then using the definition on p.4 of the notes to say what this implied about a, b. We both wrote this part independently.

(3) I got help with Problem 3 from Sam L.J. He suggested first trying some examples for each part to decide whether the statement was true or false. To prove $ab > 0$ and $bc > 0$ implies $ac > 0$ he suggested considering $(ab)(bc)$.

(4) Dr.G. I found the solution to Problem 4 on line at
`yonks.al.edu/joeblogs/absalg/hw1-sol-fall1993.html`
My apologies — I will accept a zero on this problem.

TOTAL POSSIBLE: 10 pts

(1) [3 pts] Find integers x, y such that $1 = x \cdot 13 + y \cdot 15$. Show that there are infinitely many such pairs of integers x, y.

(2) [2 pts] Let $a,\ b,\ m \in \mathbb{Z}$ with $m \geq 1$. Prove that if $a \pmod m = b \pmod m$ then $a \equiv b \pmod m$. [See p.5 of ONLINE notes.]

(3) [3 pts] Let S be the set of real numbers. For $a, b \in S$ define $a \sim b$ if $ab > 0$. For the following questions either give a proof or a counterexample.

   (i) Is the relation $\sim$ reflexive?

   (ii) Is the relation $\sim$ symmetric?

   (iii) Is the relation $\sim$ transitive?

(4) [2 pts] Let $\mathbb{R}$ be the set of real numbers. For $a, b \in \mathbb{R}$ define $a \sim b$ if $a - b + \sqrt{2}$ is irrational. Determine whether $\sim$ is an equivalence relation on $\mathbb{R}$.