SPRING 2018 – MAS 4301 — ABSTRACT ALGEBRA — HOMEWORK 1

NAME: ______________________________

ACKNOWLEDGEMENT OF OUTSIDE HELP: (List anyone that helped with this assignment. List in other books or websites you used. It is OK to get help from anyone in the class and from Dr.G. It is not ok to use someone else's work without proper citation.)

INSTRUCTIONS:

• Due Friday, January 19, 2018 either at the beginning of class or by 4pm at LIT 408.

• Staple this cover-sheet (or reproduction) to your homework.

• Write in complete sentences. All work should be handwritten and neat. Write in such a way that any student in the class can follow your work. Use examples from class and the textbook as models for your work.

• **Solutions versus answers:** Solutions, rather than answers, are expected for all problems. Even for non-proof problems, an answer alone (such as "23", "yes", etc) is not sufficient; you need to show how you arrived at the answer.

• **About proofs:** Many of the problems require formal proofs. Proofs must be properly written up, with correct mathematical notation and terminolgy, and in complete sentences. Use the examples from class and the textbook as models for your own proofs. In your proofs, you can use any result covered in class, the notes or the textbook. When using such a result, say so and cite the specific result used (e.g., "by Theorem 6.2(1) in Gallian" or "by a result from class in Chapter 6 "). • Solutions to be problems should be written in a proper and coherent manner. Explain your easoning. All work should be handwritten and neat. Write in such a way that any student in the class can follow your work. Use examples from class and the textbook as models for your work.

• **Getting help:** You may use the textbook, class notes and get help from other class members as well as Dr.G. Help is basically restricted to discussion. Any other help such as other books, websites or people must be properly cited. Of course copying someone else's work is not permitted.

TOTAL POSSIBLE: 10 pts

(1) [3 pts] Find integers x, y such that $1 = x \cdot 11 + y \cdot 13$. Show that there are infinitely many such pairs of integers x, y.

(2) [2 pts] Let a, b, $m \in \mathbb{Z}$ with $m \geq 1$. Prove that if $a \pmod{m} = b \pmod{m}$ then $a \equiv b \pmod{m}$. [See p.5 of ONLINE notes.]

(3) [3 pts] Let S be the set of positive integers. For a, $b \in S$ define $a \sim b$ if $\gcd(a, b) = 1$. For the following questions either give a proof or a counterexample.

(i) Is the relation $\sim$ reflexive?

(ii) Is the relation $\sim$ symmetric?

(iii) Is the relation $\sim$ transitive?

(4) [2 pts] Let $\mathbb{R}$ be the set of real numbers. For a, $b \in \mathbb{R}$ define $a \sim b$ if $a - b \in \mathbb{Z}$. Prove that $\sim$ is an equivalence relation.