Chapter 1 – Complex Numbers.

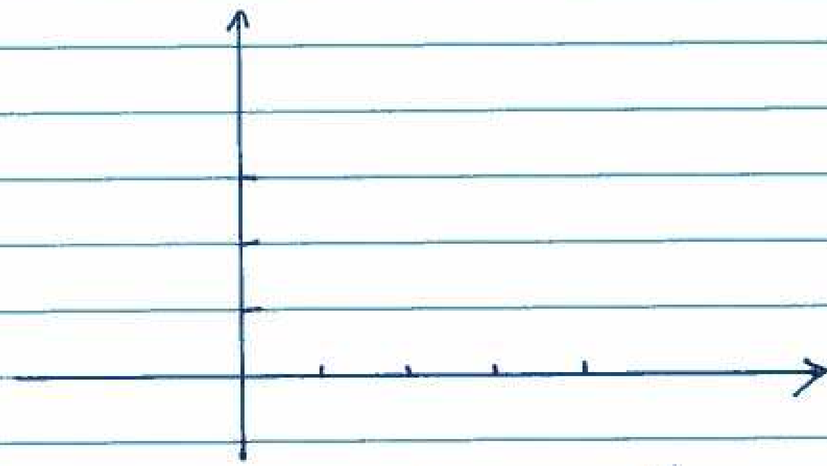
A complex number z has the form

$$z =$$

where

We can associate a complex number $z =$

with



Complex numbers are added in the ______.

If $z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$ (where)

then $z_1 + z_2 :=$.

The product is defined as

$$z_1 z_2 =$$

$$=$$

$$=$$

Properties Let z_1, z_2, z_3 be complex numbers. Then

(1) $z_1 + z_2 =$

(2) $z_1 z_2 =$

(3) $z_1 (z_2 + z_3) =$

(4) $z_1 + \quad =$

(5) $\quad = z_1$

(6) $z + (-z) = 0$,

where

(7) The set of complex numbers is

(8) $z_1 + z_2 + z_3 =$

$z_1 z_2 z_3 =$

Equality Two complex numbers

$z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$ (where)

are equal if and only if .

Theorem If $z \neq$ is a complex number

then there is a complex number z^{-1} such that .

Proof Let $z = x + iy$ where .

Suppose $z \neq$. Then

Observe that

$z \cdot (\qquad) =$

So $z \cdot \qquad = 1$,

and

$z^{-1} =$. $\square$

Example Let $z = 3 + 4i$. Find z^{-1}.

Division If z_1, z_2 are complex numbers & ___

then $\frac{z_1}{z_2} :=$

Note: If $z \neq 0$, then $\frac{1}{z} =$

Real Part & Imaginary Part

Let $z = x + iy$ where $x, y \in \mathbb{R}$.

x is called the

y is called the

We write

Example Let $z = 2/3 + 11i$.

Exercise Prove Property (2).

If z_1, z_2 are complex numbers then $z_1 z_2 = z_2 z_1$.

Proof: Let z_1, z_2 be complex numbers. Then

$z_1 =$, $z_2 =$

for

Exercise Prove Property (3).

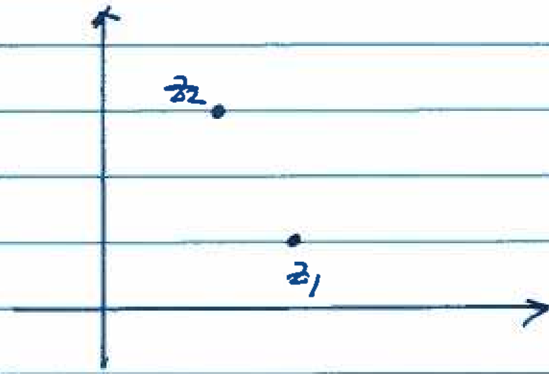
If z_1, z_2, z_3 are complex number then

$$z_1(z_2+z_3) = z_1 z_2 + z_1 z_3.$$

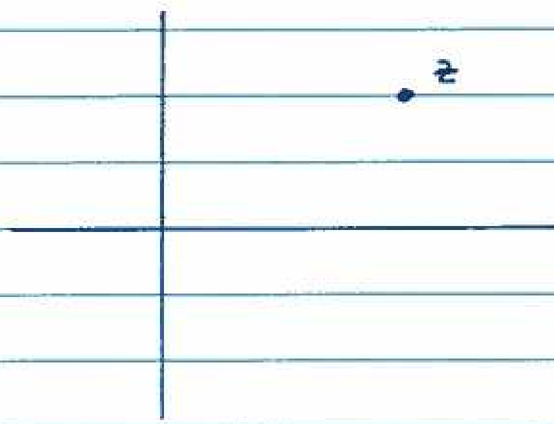
PROOF:

Geometric Interpretation of Complex Addition.

Addition of complex numbers corresponds to ______
__________.



Definition Let $z = x + iy$ where $x, y \in \mathbb{R}$.
The conjugate of z denoted by ___ is defined by



Note (1) $z\bar{z} =$ & $\bar{\bar{z}} =$

(2) The distance from z to 0 is

Definition Let $z = x + iy$ where $x, y \in \mathbb{R}$.
The modulus (or) of z is defined by

$|z| :=$

Note $|z|^2 =$ and $|\bar{z}| =$

Example Let $z = 3+4i$. Find $\bar{z}$ and $|z|$.

Properties Let z_1, z_2, z be complex numbers. Then

(1) $\overline{z_1 + z_2} =$

(2) ~~$z_1 z_2$~~ $\overline{z_1 z_2} = \overline{}$

(3) If $z_2 \neq 0$ then $\overline{\left(\frac{z_1}{z_2}\right)} =$

(4) $Re(z) =$ $\qquad\qquad$ $Im(z) =$

(5) $|z_1 z_2|$

(6) $Re(z)$

(7) $Im(z)$

(8) $|z_1 + z_2|$

PROOF: Let $z = x+iy$, $z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$ where $x, y, x_1, y_1, x_2, y_2 \in \mathbb{R}$

(1) $z_1 + z_2 =$

$\overline{z_1 + z_2} =$

(2) $z_1 z_2 =$

$\overline{z_1 z_2} =$

(3) Suppose $z_2 \neq 0$. Then

$$\left(\frac{z_1}{z_2}\right) z_2 =$$

(4)

$$z =$$

$$\bar{z} =$$

$$z + \bar{z} =$$

$$z - \bar{z} =$$

(5)

$$|z_1 z_2|^2 =$$

(6)

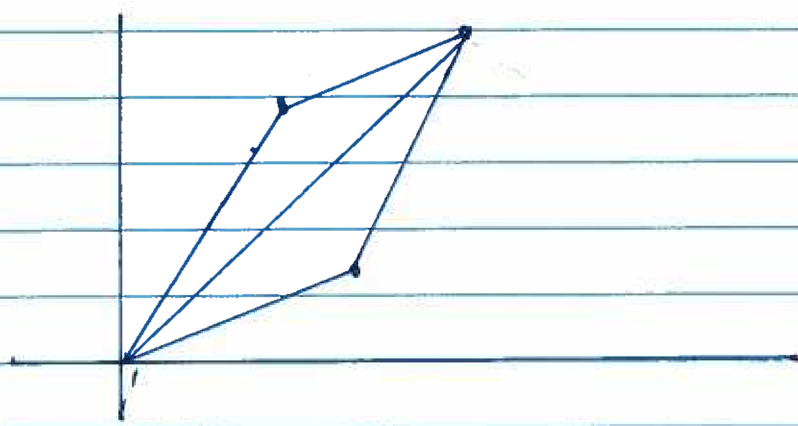
$$Re(z) =$$

(7)

$$Im(z) =$$

(8) $\left| z_1 + z_2 \right|^2$

TRIANGLE INEQUALITY



The Polar Form of a Complex Number

Let $z = x+iy$ $(x,y \in \mathbb{R})$ & $z \neq 0$.

Then the polar form of z is

where

$x =$

$y =$

$r =$

$\tan\theta =$

NOTE: Given $z \in \mathbb{C}$ (The set of Complex Numbers), $z \neq 0$,

r is

but θ is

Each value of θ is called ______________,

and is denoted by _______.

The principal value of arg z denoted by _____

is the

Example Find the polar form of $z = -\sqrt{3} + i$.

Example Let $z = -1 - i$. Find $\operatorname{Arg} z$.

Proposition

Suppose $z = r(\cos\theta + i\sin\theta)$,

$w = \rho(\cos\varphi + i\sin\varphi)$

where

Then

$zw =$

PROOF:

Corollary If $z_1, z_2 \neq 0$ then

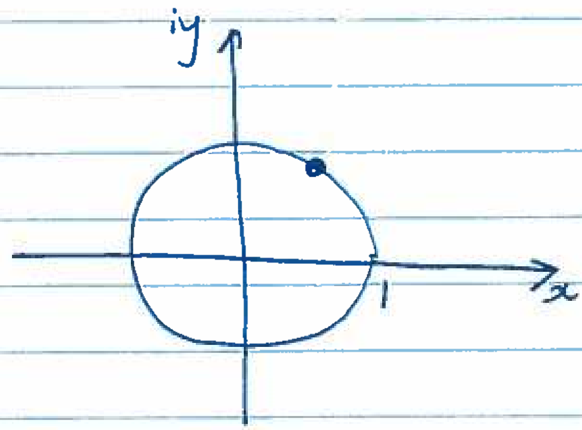
(i) $\arg(z_1 z_2) =$

(ii) $\arg\left(\frac{z_1}{z_2}\right) =$

Exponential Form

Definition For $\theta \in \mathbb{R}$, define

$$e^{i\theta} :=$$



$$|e^{i\theta}| =$$

Proposition: Let $\theta, \varphi \in \mathbb{R}$. Then

$$e^{i(\theta+\varphi)} =$$

PROOF Assume $\theta, \varphi \in \mathbb{R}$. Then

$$e^{i(\theta+\varphi)} =$$

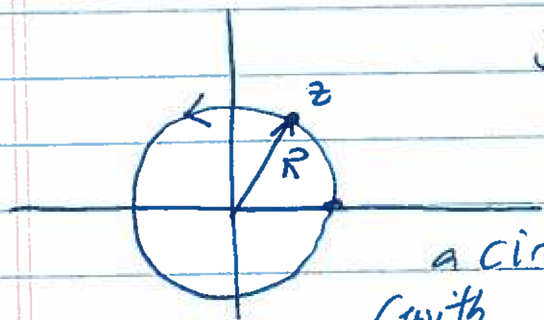
Proposition Let $\theta \in \mathbb{R}$. If $z = e^{i\theta}$ then

z __ __ __ __ and $\frac{1}{z} =$.

Proof Let $\theta \in \mathbb{R}$. Then

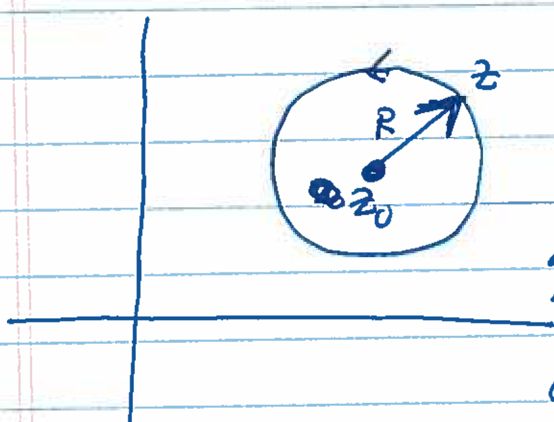
$$e^{i\theta} e^{i(-\theta)} =$$

Proposition Let $r_1, r_2, \theta_1, \theta_2 \in \mathbb{R}$ with $r_1, r_2 > 0$. Let $z_1 = r_1 e^{i\theta_1}$, $z_2 = r_2 e^{i\theta_2}$. Then $z_1 = z_2$ if and only if



The equation
$z =$
is the parametric equation of a circle center __ and radius ___ (with _____ orientation)

Note $|z| =$



The equation
$z =$
is the parametric equation of a circle center __ and radius __ (with ______ orientation)

NOTE $|z - \quad| =$

De Moivre's Theorem Let $\theta \in \mathbb{R}$, n be a positive integer. Then

$$(\cos\theta + i\sin\theta)^n =$$

Proof Let $\theta \in \mathbb{R}$. We prove the statement by ______ ________. The statement is

Ex Find $(1+i)^{100}$

Ex Find all $z \in \mathbb{C}$ that satisfy $z^3 = 1$

Theorem Let n be a positive integer. The equation

$$z^n = 1$$

has ___ complex solutions

$$z_k =$$

where $k=$

NOTE:

(1) These solutions are called __________.

(2) Let $z_1 = e^{2\pi i/n} = \omega$. Then the solutions are $z =$

Theorem Let w be a nonzero complex number with polar form

$w =$ $=$

Let n be a positive integer. Then the equation

$$z^n = w$$

has ___ complex solutions, namely

$z =$

where $k =$

Ex Solve $z^4 = -16$.

Proof of Theorem