

MAA 4402 - EXAM2 - Spring 2016

Friday, March 18

NAME:

Instructions: All work should be written in a proper and coherent manner, and in a way that any student can follow your work. Show all necessary working and reasoning.
NO calculators are allowed.

DO ALL QUESTIONS

1. [4+6=10]

(a) On what domain D is $\text{Log}(z)$ analytic?
Sketch this domain.

(b) Assuming $\text{Log}(z)$ is analytic on D , show that
$$\frac{d}{dz} \text{Log}(z) = \frac{1}{z}.$$

2. [10] Solve

$$\sin(z) = 2.$$

3. [10] Let C be the straight-line segment from $z=1$ to $z=1+2i$. Evaluate the contour integral

$$\int_C \exp(\bar{z}) dz.$$

4. [10] Let C be the arc of the circle $|z|=2$ (with positive orientation) from $z=2$ to $z=2i$. Show that

$$\left| \int_C \frac{\bar{z} + 4}{z^2 - 1} dz \right| \leq 2\pi$$

5. [15+4] Let $f(z) = \frac{\text{Log}(z+4i)}{(z+2)^2}$

- (i) Sketch the domain on which $f(z)$ is analytic.
 (ii) Evaluate $\int_C f(z) dz$ when

(a) C is the simple closed circle $|z|=1$ with positive orientation.

(b) C is the simple closed circle $|z|=3$ with positive orientation.

BONUS [2]



His father owned the pictorial windmill. Who was his famous mathematician?

Where did he go to university and at what age?