FALL 2018 – MAA 4402/5404 — COMPLEX VARIABLES — HOMEWORK 1

NAME: ________________________________

INSTRUCTIONS:

• Due Friday, August 31, 2018 either at the beginning of class or by 4pm at LIT 408.

• Staple this cover-sheet (or reproduction) to your homework.

• Write in complete sentences.

• Solutions to be problems should be written in a proper and coherent manner. Explain your reasoning. All work should be handwritten and neat. Write in such a way that any student in the class can follow your work. Use examples from class and the textbook as models for your work.

• Show all necessary work.

• **Getting help:** You may use the textbook, class notes and get help from other class members as well as Dr.G. However help is restricted to discussion only. Your final solution must be written up independently and in your own words. It is not permitted to copy someone else's work or copy solutions from anywhere. Any help on any question must explicitly cited giving details. See examples below.

DETAILED ACKNOWLEDGMENT OF OUTSIDE HELP: (Included details here or in the body of your solution)

EXAMPLES OF DETAILED ACKNOWLEDGEMENT OF OUTSIDE HELP:

(1) Me and Mary K. discussed Problem 1. We went over the problem of proving $z_1 + (z_2 + z_3) = (z_1 + z_2) + z_3$ together. Then agreed to approach Problem 1 in a similar way.

(2) I asked Dr.G. for help with Problem 2(i). Dr.G. explained how to prove that $\mathrm{Im}(iz) = \mathrm{Re}(z)$.

(3) I got help with Problem 3(c) from Sam L.J. We first sketched $\left\{z : \mathrm{Arg}(z) = -\frac{\pi}{4}\right\}$ and discussed the relation with the set $\left\{z : \mathrm{Arg}(z - i) = -\frac{\pi}{4}\right\}$.

(4) Dr.G. I found the solution to Problem 4 on line at
`yonks.al.edu/joeblogs/absalg/hw1-sol-fall1993.html`
My apologies — I will accept a zero on this problem.

(5) In Problem 4, Me and Uma T. discussed the importance of using the Property $\mathrm{Re}(z_1 + z_2) = \mathrm{Re}(z_1) + \mathrm{Re}(z_2)$.

TOTAL POSSIBLE: 10 pts

(1) [3 pts] PROVE the following property of complex numbers from the definition of complex multiplication. Use either the definition given on p.1 of the ONLINE notes, or the definition given on p.2 of the textbook.

If $z_1,\ z_2,\ z_3 \in \mathbb{C}$ then
$$z_1(z_2 z_3) = (z_1 z_2) z_3.$$

(2) [2 pts] Let $z \in \mathbb{C}$. Prove each of the following.

(i) $\mathrm{Re}(iz) = -\mathrm{Im}(z)$. (ii) If $\mathrm{Im}(z) > 0$ then $\mathrm{Im}(-1/(z + 1)) > 0$.

(3) [3 pts] Sketch the following sets of complex numbers.

(a) $\{z : |2z + 4 + 2i| > 2\}$

(b) $\{z : |\overline{z} + 1 + i| < 1\}$

(c) $\left\{z : \mathrm{Arg}(z - i) = -\frac{\pi}{4}\right\}$

(4) [2 pts] Show that
$$|\mathrm{Re}(3 + i + 2(\overline{z})^2 - i\,z)| \leq 6, \qquad \text{when } |z| \leq 1.$$