Homework 2

(I.)(a) $y^2 + x - 3 = 0 \iff y^2 = 3 - x$

which defines a function on the interval $(-\infty, 3)$

namely $y = \sqrt{3-x}$, or $y = -\sqrt{3-x}$.

$2y\frac{dy}{dx} = -1$ since $\frac{d}{dx}(y^2) = \frac{d}{dx}(3-x)$,

& $\frac{dy}{dx} = -\frac{1}{2y}$ where $y \neq 0$ since $x < 3$.

So the equation $y^2 + x - 3 = 0$ is an implicit solution to $\frac{dy}{dx} = -1/(2y)$ on $(-\infty, 3)$.

(II.) Assume $y = y(x)$ &

$e^{xy} + y = x - 1$.

Then $\frac{d}{dx}(e^{xy} + y) = \frac{d}{dx}(x-1)$,

$e^{xy}\frac{d}{dx}(xy) + \frac{dy}{dx} = 1$,

$e^{xy}\left(x\frac{dy}{dx} + y\right) + \frac{dy}{dx} = 1$

$\frac{dy}{dx}(x e^{xy} + 1) = 1 - y e^{xy}$ &

$$\frac{dy}{dx} = \frac{1 - y e^{xy}}{x e^{xy} + 1}\left(\frac{e^{-xy}}{e^{-xy}}\right) = \frac{e^{-xy} - y}{x + e^{-xy}}.$$

So the equation implicitly defines a solution of the DE.

(21)(a) Let $y = x^m$. Then $y' = mx^{m-1}$, $y'' = m(m-1)x^{m-2}$, and

$$3x^2y'' + 11xy' - 3y = 3m(m-1)x^m + 11mx^m - 3x^m$$
$$= x^m(3m(m-1) + 11m - 3)$$
$$= x^m(3m^2 + 8m - 3) = 0$$

when $3m^2 + 8m - 3 = 0$

ie $m = \dfrac{-8 \pm \sqrt{64+36}}{6}$

$= \dfrac{-8 \pm \sqrt{100}}{6} = \dfrac{-8 \pm 10}{6} = 2/6, \dfrac{-18}{6}$

$= 1/3, -3.$

So for $m = 1/3, -3$, $\phi(x) = x^m$ is a solution of the DE.

(25)

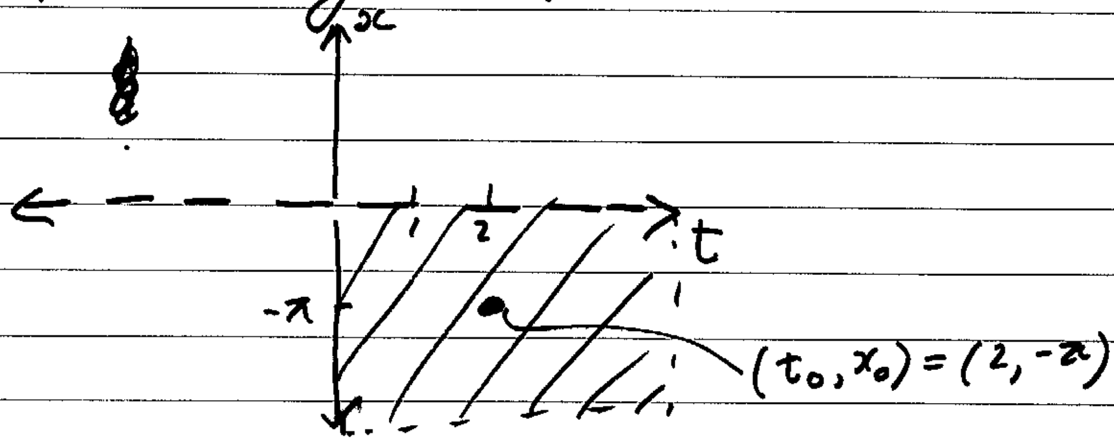
$$3x\frac{dx}{dt} + 4t = 0 \iff \frac{dx}{dt} = \frac{-4t}{3x} \quad (x \neq 0).$$

Let $f(t, x) = -\dfrac{4t}{3x}$.

Then f is continuous for $x \neq 0$.

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x} \frac{-4}{3} t x^{-1} = \frac{4t}{3x^2}$$

which is continuous for $x \neq 0$.



So $f(t,x)$ & $\frac{\partial f}{\partial x}$ are both continuous on an open rectangle containing $(t_0, x_0) = (2, -\pi)$; for example

$$R = \{(t,x) : 0 < t < 4, -2\pi < x < 0\}.$$

The Theorem implies the IVP has a unique sol'n $\phi(t)$ defined on some interval

$$2-\delta < t < 2+\delta$$

some $0 < \delta$.

(28) IVP: $\frac{dy}{dx} = 3x - \sqrt[3]{y-1}$, $y(2) = 1$.

Let $f(x,y) = 3x - \sqrt[3]{y-1}$.
$f(x,y)$ is continuous for all (x,y).

$$\frac{\partial f}{\partial y} = -\frac{1}{3}(y-1)^{-2/3}$$

which is continuous for $y \neq 1$.
$\frac{\partial f}{\partial y}$ is not continuous at $(x_0, y_0) = (2,1)$ & hence is not continuous on any open rectangle that contains $(x_0, y_0) = (2,1)$.
The Theorem does not apply and we can not deduce whether the IVP has a unique sol'n $\phi(x)$ on an open interval containing $x_0 = 2$ or not.