SPRING 2014 – MAP 2302 — ELEMENTARY DIFFERENTIAL EQUATIONS — HOMEWORK 4

NAME: ____________________________

INSTRUCTIONS:

- Due in class on Friday, January 31, 2014.
- Staple this cover-sheet to your homework.
- Write in complete sentences.
- Solutions to be problems should be written in a proper and coherent manner. All work should be handwritten and neat. Write in such a way that any student in the class can follow your work. Use examples from class and the textbook as models for your work.
- Show all necessary work.

ACKNOWLEDGEMENT: I obtained help from the following:

(1) The general solution of the seperable equation

$$f(y)\,\frac{dy}{dx} = g(x)$$

is given implicitly by

$$F(y) = G(x) + c,$$

where

c is any constant,

$F(y) = \int f(y)\,dy$,

and

$G(x) = \int g(x)\,dx$.

(b) $\frac{dy}{2\sqrt{y+1}} = \cos x \, dx$ (seperable)

$$\Leftrightarrow \int \frac{dy}{2\sqrt{y+1}} = \int \cos x \, dx + c$$

$$\frac{1}{2} \frac{(y+1)^{1/2}}{1/2} = \sin x + c$$

$$(y+1)^{\frac{1}{2}} = \sin x + c.$$

$y(\pi) = 0$ so $1 = \sin \pi + c$ & $c = 1.$

So

$$y + 1 = (\sin x + 1)^2$$

$$y = -1 + (\sin x + 1)^2$$

Soln to IVP is

$$y = \sin^2 x + 2 \sin x$$ (explicitly)

for $-\pi/2 < x < 3\pi/2$.

(2) Solve the following initial value problem.

$$\frac{dy}{dx} = 2\sqrt{y+1}\ \cos x, \qquad y(\pi) = 0.$$

Your solution y should be given explicitly in terms of x.

(3) To solve the linear equation

$$\frac{dy}{dx} + p(x)\,y = q(x) \qquad (*),$$

(i) we first let

$$\mu(x) = e^{P(x)},$$

where

$$P(x) = \int p(x)\,dx$$

_ _ _ _ _ _ _ _ .

(ii) We multiply both sides of $(*)$ by $\mu(x)$ and write the resulting equation as

$$\frac{d}{dx}\left(\mu(x)\,y\right) = \mu(x)\,q(x).$$

(iii) The general solution of $(*)$ is given by

$$y = \frac{1}{\mu(x)}\left(\int \mu(x)\,q(x)\,dx + C\right)$$

where C is any constant.

(4)

(i) A first order DE is <u>exact</u> if it can be written in the form

(*) $\frac{d}{dx}\left(F(x,y)\right) = 0$,

where

$y = y(x)$.

(ii) (*) is equivalent to the DE

(**) $M(x,y) + N(x,y)\frac{dy}{dx} = 0$

where

$M(x,y) = \frac{\partial F}{\partial x}$,

$N(x,y) = \frac{\partial F}{\partial y}$.

(iii) The general solution of (*) is given implicitly by

$F(x,y) = c$

where c is any constant.

(iv) If the 1st order partial derivatives of $M(x,y)$, $N(x,y)$ are continuous and (**) is exact, then

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}.$$

(5) <u>Theorem</u> If the 1st order partial derivatives of $M(x,y)$ and $N(x,y)$ are continuous on a open rectangle R, and

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

for all (x,y) in R, then the DE

$$M(x,y) + N(x,y)\frac{dy}{dx} = 0$$

is exact on R.

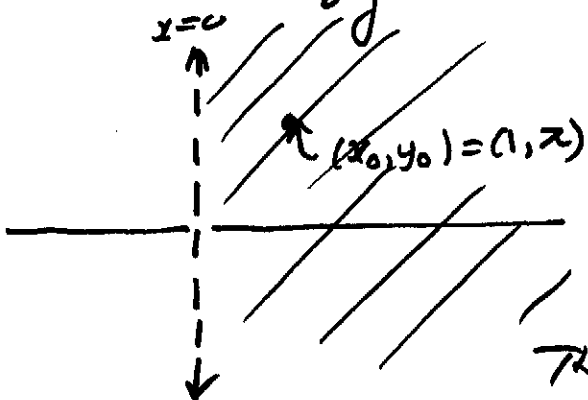
(6) Solve the IVP

$$\left(\frac{1}{x} + 2y^2x\right) + \left(2yx^2 - \cos(y)\right)\frac{dy}{dx} = 0, \quad y(1) = \pi.$$

Let $M(x,y) = \frac{1}{x} + 2y^2x$,

$N(x,y) = 2yx^2 - \cos y$.

Then $\frac{\partial M}{\partial y} = 4yx = \frac{\partial N}{\partial x}$ for $x \neq 0$.



Since 1st order partial derivatives of M, N are continuous for $x>0$ and $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ for $(x,y) \in R$, open rectangle $R = \{(x,y) : x > 0\}$.

The DE is exact on R & there is a function $F(x,y)$ such that

$$\frac{\partial F}{\partial x} = M = \frac{1}{x} + 2y^2x, \quad \& \quad \frac{\partial F}{\partial y} = N = 2yx^2 - \cos y.$$

$$F = \int \left(\frac{1}{x} + 2y^2x\right) dx$$

$$= \ln x + x^2y^2 + k(y).$$

$$\frac{\partial F}{\partial y} = 2x^2y + k'(y) = 2yx^2 - \cos y,$$

$$k'(y) = -\cos y, \quad k = -\int \cos y \, dy = -\sin y,$$

& we take

$$F(x,y) = \ln x + x^2y^2 - \sin y.$$

The general sol<u>n</u> is given implicitly by

$$\ln x + x^2y^2 - \sin y = c.$$

$y(1) = \pi$ so

$$\ln 1 + \pi^2 - \sin \pi = c,$$

$$c = \pi^2$$

The sol<u>n</u> to the IVP is given implicitly by

$$\ln x + x^2y^2 - \sin y = \pi^2.$$