SPRING 2014 – MAP 2302 — ELEMENTARY DIFFERENTIAL EQUATIONS — HOMEWORK 4

NAME: _____

INSTRUCTIONS:

- Due in class on Friday, January 31, 2014.
- Staple this cover-sheet to your homework.
- Write in complete sentences.
- Solutions to be problems should be written in a proper and coherent manner. All work should be handwritten and neat. Write in such a way that any student in the class can follow your work. Use examples from class and the textbook as models for your work.
- Show all necessary work.

ACKNOWLEDGEMENT: I obtained help from the following:

(1) The general solution of the seperable equation

$$f(y)\frac{dy}{dx} = g(x)$$

is given implicitly by

$$F(y) = G(x) + \text{----},$$

where

c is ---------,

F(y) =

---------)

and

G(x) =

---------.

(2) Solve the following initial value problem.

$$\frac{dy}{dx} = 2\sqrt{y+1}\cos x, \qquad y(x)=0.$$

Your solution y should be given explicitly in terms of x.

(3) To solve the linear equation

$$\frac{dy}{dx} + f(x)y = g(x) \qquad (*),$$

(i) we first let

$$\mu(x) = e^{P(x)},$$

where

$$P(x) = \text{\_\_\_\_\_}$$

(ii) We multiply both sides of $(*)$ by _____ and write the resulting equation as

$$\frac{d}{dx}\left(\text{\_\_\_\_\_}\right) = \text{\_\_\_\_\_}$$

(iii) The general solution of $(*)$ is given by

$$y = \text{\_\_\_\_\_}$$

where

_____

(4)

(i) A first order DE is <u>exact</u> if it can be written in the form

(*) $\frac{d}{dx}(\_\_\_\_\_) = \_\_\_\_\_,$

where

$y = \_\_\_\_\_.$

(ii) (*) is equivalent to the DE

(**) $M(x,y) + N(x,y)\frac{dy}{dx} = 0$

where

$M(x,y) = \_\_\_\_,$

$N(x,y) = \_\_\_\_\_.$

(iii) The general solution of (*) is given _____ by

_____ = ___

where ___ is _____.

(iv) If the _____ of $M(x,y)$, $N(x,y)$ are _____ and (**) is exact, then

$\frac{\partial M}{\partial \_\_} = \frac{\partial N}{\partial \_\_}.$

(5) Theorem If the _____ of $M(x,y)$ and $N(x,y)$ are continuous on a _____ R, and

$$\frac{\partial M}{\partial \_\_} = \frac{\partial N}{\partial \_\_}$$

for all (x,y) in R, then the DE

$$M(x,y) + N(x,y)\frac{dy}{dx} = 0$$

is _____.

(6) Solve the IVP

$$\left(\frac{1}{x} + 2y^{3x}\right) + \left(2yx^2 - \cos(y)\right)\frac{dy}{dx} = 0, \quad y(1) = \pi.$$