Homework 5

(1) Let $y_1 = e^{5x}$, then $y_1' = 5e^{5x}$ & $y_1'' = 25e^{5x}$ &

$L[y_1] = 25e^{5x} + p_1(x)\, 5e^{5x} + p_2(x)\, e^{5x}$

$= e^{5x}(25 + p_1(x)\,5 + p_2(x)) = 0$

if $p_2(x) = -25 - p_1(x)\,5$.

For example, let $p_1(x) = 1$, $p_2(x) = -30$ then.

$L[y_1] = 0$ if

$L[y] = y'' + y' - 30y$.

(3)

(a) Let $y_1 = e^{2x}$. Then $y_1' = 2e^{2x}$, $y_1'' = 4e^{2x}$ &

$xy_1'' - (2x+2)y_1' + 4y_1 = 4xe^{2x} - (2x+2)\,2e^{2x} + 4e^{2x}$

$= e^{2x}(4x - 4x - 4 + 4) = 0 \cdot e^{2x} = 0$ &

$y_1 = e^{2x}$ is a soln.

Let $y_2 = 2x^2+2x+1$. Then $y_2' = 4x+2$, $y_2'' = 4$ &

$xy_2'' - (2x+2)y_2' + 4y_2 = 4x - (2x+2)(4x+2) + 4(2x^2+2x+1)$

$= 4x - (8x^2 + 12x + 4) + 8x^2 + 8x + 4$

$= -8x^2 + 8x^2 - 8x + 8x - 4 + 4 = 0$, &

$y_2 = 2x^2+2x+1$ is a soln.

(b) $W[y_1, y_2] = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{2x} & 2x^2+2x+1 \\ 2e^{2x} & 4x+2 \end{vmatrix}$

$= e^{2x}(4x+2) - 2e^{2x}(2x^2+2x+1)$

$= e^{2x}(4x+2 - 4x^2 - 4x - 2) = -4x^2 e^{2x}$.

(c) y_1, y_2 are linearly indept. on $(0, \infty)$
since $W[y_1, y_2] \neq 0$ on $(0, \infty)$.

(d) Since y_1, y_2 are linearly indept. solution the general soln is given by

$$y = c_1 y_1 + c_2 y_2$$
$$= c_1 e^{2x} + c_2 (2x^2 + 2x + 1)$$

for any constants c_1, c_2.

(e) Let $y = c_1 e^{2x} + c_2 (2x^2 + 2x + 1)$.
Then

$$y' = 2c_1 e^{2x} + c_2 (4x + 2).$$

$$y(1) = c_1 e^2 + 5c_2 = e^2 - 10$$
$$y'(1) = 2c_1 e^2 + 6c_2 = 2e^2 - 12.$$

Clearly the soln is $c_1 = 1$, $c_2 = -2$.
Hence

$$y = e^{2x} - 2(2x^2 + 2x + 1)$$

is the soln.

(5) We consider two DEs

(A) $y'' - 2y' - 3y = t e^{3t}$,
(B) $y'' - 2y' - 3y = t^2 \cos 3t - 5 \sin 3t$.

A.E: $(r^2 - 2r - 3) = 0$
$(r - 3)(r + 1) = 0$
$r = 3, -1$.

Since $r=3$ is a single root of the A.E. a particular soln of (A) has the form

$$y_p = t(A_0 + A_1 t)\,e^{3t}.$$

Since $r=3i$ is not a root of the A.E. a particular soln of (B) has the form

$$y_p = (B_0 + B_1 t + B_2 t^2)\cos 3t + (C_0 + C_1 t + C_2 t^2)\sin 3t.$$

Hence by the Superposition Principle a particular soln of the DE has the form

$$y_p = t(A_0 + A_1 t)\,e^{3t} + (B_0 + B_1 t + B_2 t^2)\cos 3t + (C_0 + C_1 t + C_2 t^2)\sin 3t$$

for some constants $A_0, A_1, B_0, B_1, B_2, C_0, C_1, C_2$.

(6) $w(x) = 2x + 1 = 0$ for $x = -1/2$ which is in $(-1,1)$. Hence $w(x)$ could not be the wronskian of two solns y_1, y_2. Either the wronskian $W[y_1,y_2]$ is never zero on $(-1,1)$ or $W[y_1,y_2](x) = 0$ for all x in $(-1,1)$.