Answers to Practice Problem Set 2

1. (a) $\frac{1}{6}t - \frac{5}{36}$
   (b) te^{-2t}
   (c) $3(\frac{1}{6}t - \frac{5}{36}) + 4te^{-2t}$

2. (a) $\cot t \cos t - \frac{1}{2}\csc t$
   (b) $y_1(t) = t^{-1}$ and $y_2(t) = t^{-4}$

3. (a) $y_2(t) = -\frac{1}{2}t^{-1}e^{-2t}$
   (b) $y = 7y_2(t)$ and $y = 7y_2(t) + y_1(t)$, for instance.

4. (a) $\frac{1}{2}y'' + 2y' + 4y = 0$, $y(0) = 3$, $y'(0) = 0$
   (b) $y(t) = 3e^{-2t}\cos 2t + 3e^{-2t}\sin 2t$
   (c) $y(t) = 3\sqrt{2}e^{-2t}\sin(2t + \frac{1}{4}\pi)$

5. (a) $2y'' + 10y' + 12y = 0$, $y(0) = 0$, $y'(0) = 4$
   (b) $y(t) = 4e^{-2t} - 4e^{-3t}$

6. (a) $y_1(t) = \frac{1}{2}t^2 - \frac{3}{2}t + \frac{7}{4}$
   (b) $y_2(t) = -te^{-2t}$
   (c) $y_p(t) = 2t^2 - 6t + 7 - 5te^{-2t}$

7. (a) $y_p(t) = -\frac{1}{9} + \frac{1}{9}\sin 3t \cdot \ln|\sec 3t + \tan 3t|$
   (b) $y_2(t) = te^t$

8. (a) $\frac{1}{2}y'(t)^2 - \frac{1}{6}y^6 = C$
   (b) $y(t) = -\frac{3}{4}e^{-8t}\cos 8t - \frac{3}{4}e^{-8t}\sin 8t$

9. $\frac{1}{2}(y')^2 - \frac{1}{3}y^3 = K$

10. (a) $y_p(t) = \frac{1}{2}t + \frac{3}{4}$
    (b) $y_p(t) = -te^t$
    (c) $y_p(t) = 7(\frac{1}{2}t + \frac{3}{4}) + 9(-te^t)$

11. (a) $y_p(t) = (\ln|\cos t|)\cos t + t\sin t$
    (b) $\{t^2, t^{-3}\}$

12. $y(t) = e^{-t}\cos 2t + \frac{1}{2}e^{-t}\sin 2t$

13. (a) No
    (b) $\phi_1(t) = 2t$, $\phi_2(t) = 3t$

14. $\frac{1}{2}(y')^2 + \frac{1}{2}\cos 2y = K$

15. (a) $k_1 e^{2x} + k_2 e^{4x}$
    (b) $\frac{1}{2} - 3xe^{2x} + k_1 e^{2x} + k_2 e^{4x}$

16. (a) $3y'' + 12y = 6e^{-t}$, $y(0) = y'(0) = 0$
    (b) $y(t) = \frac{2}{5}e^{-t} - \frac{2}{5}\cos 2t + \frac{1}{5}\sin 2t$