MAP 2302—Practice Problem Set #2

1. Find a particular solution to each of the following ODEs.

   (a) $y'' + 5y' + 6y = t$

   (b) $y'' + 5y' + 6y = e^{-2t}$

   (c) $y'' + 5y' + 6y = 3t + 4e^{-2t}$

2. (a) Use the method of variation of parameters to find a particular solution to the ODE $y'' + y = \csc^3 t$.

   (b) Find two linearly independent solutions to the ODE $y'' + 6t^{-1}y' + 4t^{-2}y = 0$ for $t > 0$.

3. (a) Consider the ODE $ty'' + (2 + 2t)y' + 2y = 0$ for $t > 0$. One solution is $y_1(t) = t^{-1}$. Use the method of reduction of order to find another solution $y_2(t)$ which is linearly independent from $y_1(t)$.

   (b) Suppose $y_1(t)$ is a nonzero solution to the ODE $y'' + ty = 0$ and $y_2(t)$ is a solution to the ODE $y'' + ty = e^t$. Express two different solutions to the ODE $y'' + ty = 7e^t$ in terms of $y_1(t)$ and $y_2(t)$.

4. A mass of $\frac{1}{2}$ kg is attached to a spring with stiffness 4 N/m; the damping constant for the system is 2 N-sec/m. The mass is pulled 3 m to the right of equilibrium and is then released from rest at time $t = 0$.

   (a) Write down an initial value problem satisfied by the position function $y(t)$ of the object.

   (b) Find the position $y(t)$ of the mass as a function of t by solving this IVP.

   (c) Express the position function from (b) in the form $y(t) = Ae^{\alpha t}\sin(\beta t + \phi)$.

5. A ball of mass 2 is attached to a spring whose spring constant is 12. The force of friction on the ball is given by the damping coefficient 10. The ball starts at its equilibrium point with an initial velocity of 4.

   (a) Let $y(t)$ denote the position of the ball at time t. Find a differential equation and an initial condition satisfied by $y(t)$.

   (b) Find $y(t)$ by solving the initial value problem from (a).

6. Find a particular solution to each of the following ODEs.

   (a) $y'' + 3y' + 2y = t^2$

   (b) $y'' + 3y' + 2y = e^{-2t}$

   (c) $y'' + 3y' + 2y = 4t^2 + 5e^{-2t}$

7. (a) Use the method of variation of parameters to find a particular solution to the ODE $y'' + 9y = \sec^2 3t$.

(b) One solution to the ODE $t^2y'' - t(t+2)y' + (t+2)y = 0$ is given by $y_1(t) = t$. Use the method of reduction of order to find another linearly independent solution $y_2(t)$. (No credit for solutions based on guessing.)

8. (a) Write down the equation that the Energy Integral Lemma gives for solutions to the ODE $y'' = y^5$. (You should not attempt to solve the resulting ODE.)

   (b) A mass of $\frac{1}{8}$ kg is attached to a spring with stiffness 16 N/m; the damping constant for the system is 2 N-sec/m. The mass is pulled $\frac{3}{4}$ m to the left of equilibrium and is then released from rest at time $t = 0$. Find the position of the mass as a function of t.

9. Write down the equation that the Energy Integral Lemma gives for solutions to the ODE $y'' = y^2$. (You should not attempt to solve the resulting ODE.)

10. Find a particular solution to each of the following ODEs.

    (a) $y'' - 3y' + 2y = t$

    (b) $y'' - 3y' + 2y = e^t$

    (c) $y'' - 3y' + 2y = 7t + 9e^t$

11. (a) Use the method of variation of parameters to find a particular solution to the ODE $y'' + y = \sec t$.

    (b) Find a fundamental set of solutions to the ODE $t^2y'' + 2ty' - 6y = 0$ for $t > 0$.

12. A mass of 3 kg is attached to a spring with stiffness 15 N/m; the damping constant for the system is 6 N-sec/m. The mass is pulled 1 m to the right and is then released from rest at time $t = 0$. Find the position of the mass as a function of t.

13. Short answer. No explanations needed.

    (a) Suppose $y = \phi_1(x)$ and $y = \phi_2(x)$ are solutions to the nonhomogeneous ODE $y'' - x^3y = e^x$. Is $y = \phi_1(x) + \phi_2(x)$ also a solution to this ODE?

    (b) Write down two functions $\phi_1(x)$, $\phi_2(x)$ which are *not* linearly independent.

14. Write down the equation that the Energy Integral Lemma gives for solutions to the ODE $y'' = \sin 2y$.

15. Find the general solution to each of the following differential equations.

    (a) $y'' - 6y' + 8y = 0$

    (b) $y'' - 6y' + 8y = 6e^{2x} + 4$

16. An object with mass 3 is attached to a spring with spring constant 12. There is no damping, but an external force given by the function $f(t) = 6e^{-t}$ acts on the object. The object starts at rest in the equilibrium position at time $t = 0$.

(a) Write down an initial value problem satisfied by the position function $y(t)$ of the object.

(b) Find the equation of motion of the object by solving the IVP from (a).