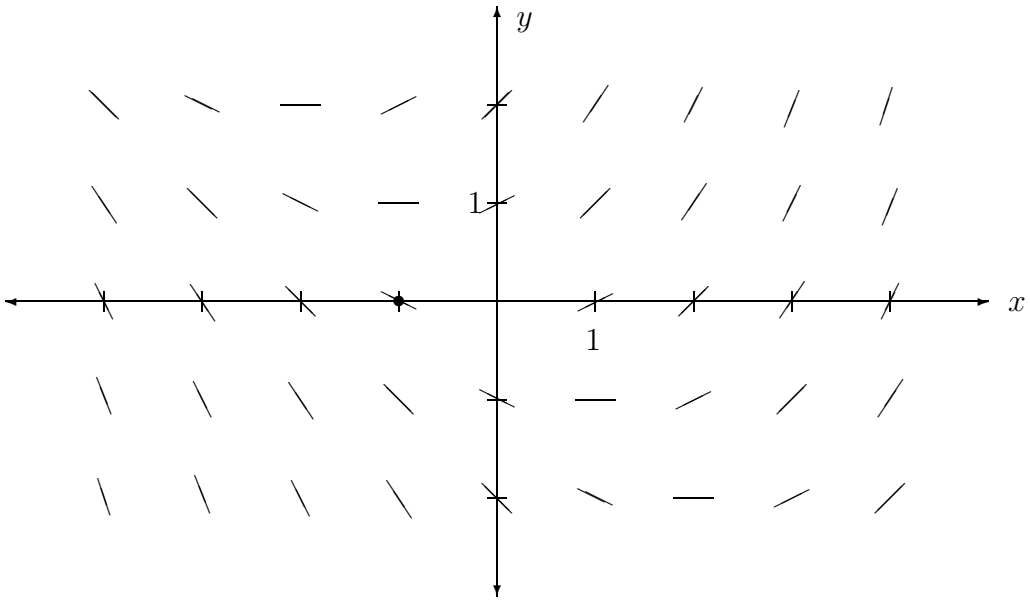
MAP 2302—Practice Problem Set #1

1. (a) i. Determine whether $x = \cos 2t$ is a solution to the ODE $\dfrac{dx}{dt} + tx = \sin 2t$.

   ii. Determine whether $y - \ln y = x^2 + 1$ an implicit solution to the ODE $\dfrac{dy}{dx} = \dfrac{2xy}{y-1}$.

   (b) The direction field for a differential equation is given below. Sketch the solution to this differential equation which passes through the point $(-1, 0)$.



2. (a) Solve the initial value problem below. Your solution should express y explicitly as a function of t.

$$t^{-1}\frac{dy}{dt} = 2\cos^2 y, \quad y(0) = \pi/4$$

   (b) Find the general solution to the ODE $(x^2+1)\dfrac{dy}{dx} + xy - x = 0$. Your solution should express y explicitly as a function of x.

3. (a) Find the general solution to the ODE $(2x + y)\,dx + (x - 2y)\,dy = 0$. Your solution should express y explicitly as a function of x.

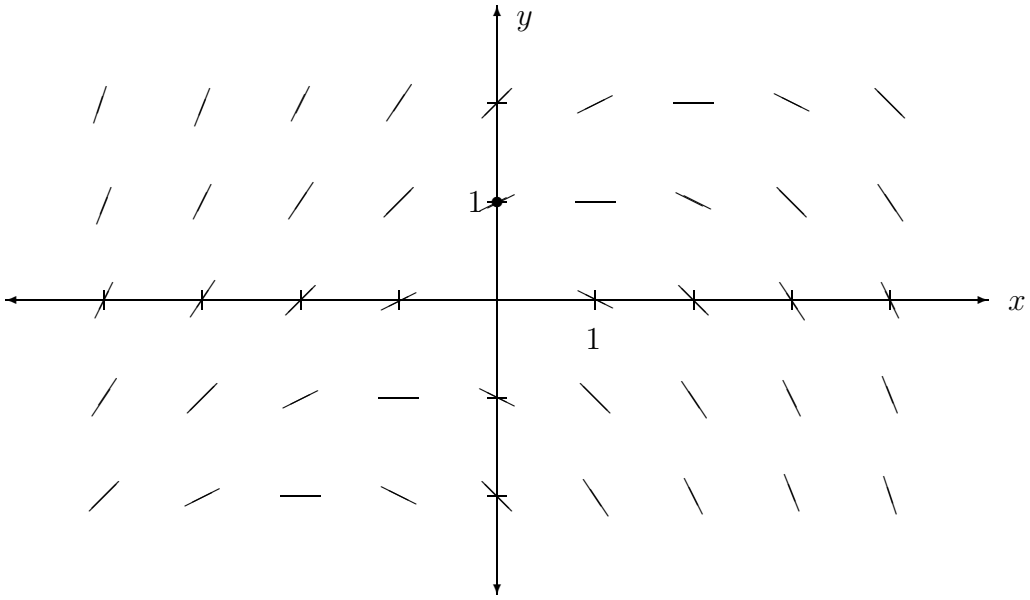
   (b) Find the general solution to the ODE $\dfrac{dy}{dx} = 2x^{-1}y - x^2y^2$. Your solution should express y explicitly as a function of x.

4. (a) Short answer. No explanations needed.

   i. Is the ODE $3x\,dx + 5y^2\,dy = 0$ exact?

   ii. Give an example of a third-order linear ODE.

   (b) The direction field for a differential equation is given below. Sketch the solution to this differential equation which passes through the point $(0, 1)$.



5. (a) Find the general solution to the ODE $\dfrac{dy}{dx} = \dfrac{\sec^2 y}{1+x^2}$. An implicit solution is enough here; you don't need to solve for y in terms of x.

   (b) Find the general solution to the ODE $x\dfrac{dy}{dx} + 2y = x^{-3}$. Your solution should express y explicitly as a function of x.

6. (a) Solve the initial value problem below. Your solution should express y explicitly as a function of t.

$$(e^t y + te^t y)\,dt + (te^t + 2)\,dy = 0, \quad y(0) = -1$$

   (b) Find the general solution to the ODE $\dfrac{dy}{dx} = \sqrt{x+y} - 1$. An implicit solution is enough here; you don't need to solve for y in terms of x.

7. (a) Find the general solution to the ODE $\dfrac{dy}{dx} = \dfrac{x^2 - y^2}{3xy}$. An implicit solution is enough here; you don't need to solve for y in terms of x.

   (b) Find the general solution to the ODE $(x^4 - x + y)\,dx - x\,dy = 0$. An implicit solution is enough here; you don't need to solve for y in terms of x.

8. (a) Solve the initial value problem below. Your answer should express y explicitly as a function of x.

$$\frac{dy}{dx} - \frac{y}{x} = xe^x, \qquad y(1) = e - 1.$$

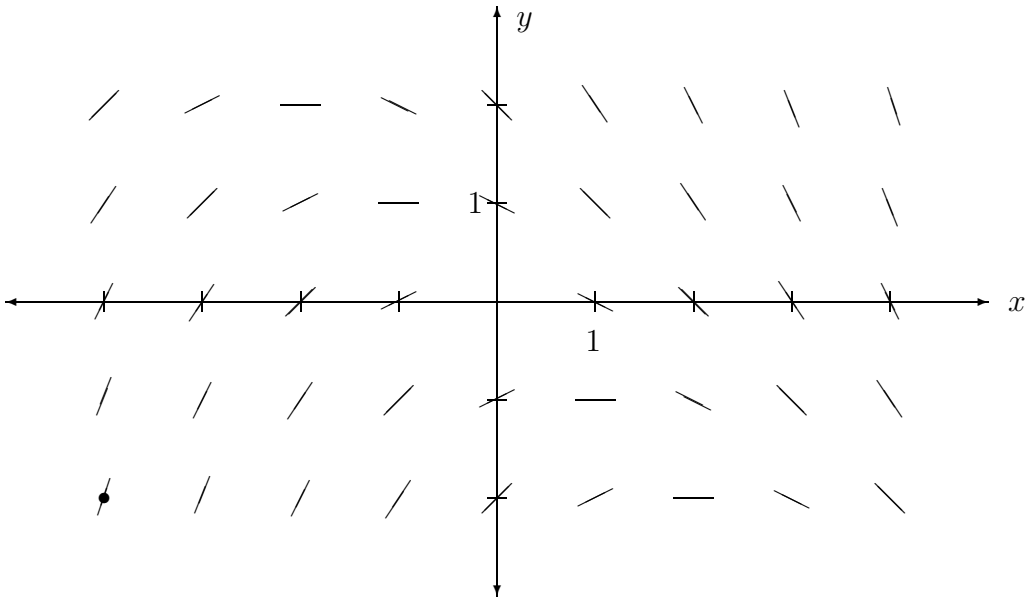
   (b) Find the general solution to the differential equation $\dfrac{dy}{dx} = \dfrac{xy + y^2}{x^2}$. An implicit solution is enough here; you don't need to solve for y in terms of x.

9. (a) Find the general solution to the differential equation

$$\frac{dx}{y} + \left(y^3 - \frac{x}{y^2}\right) dy = 0.$$

An implicit solution is enough here; you don't need to solve for y.

(b) The direction field for a differential equation is given below. Sketch the solution to this differential equation which passes through the point $(-4, -2)$.



10. (a) Solve the initial value problem below. Your answer should express y explicitly as a function of x.

$$y' + 2x^{-1}y = x^{-1} + 3 \qquad y(1) = 2$$

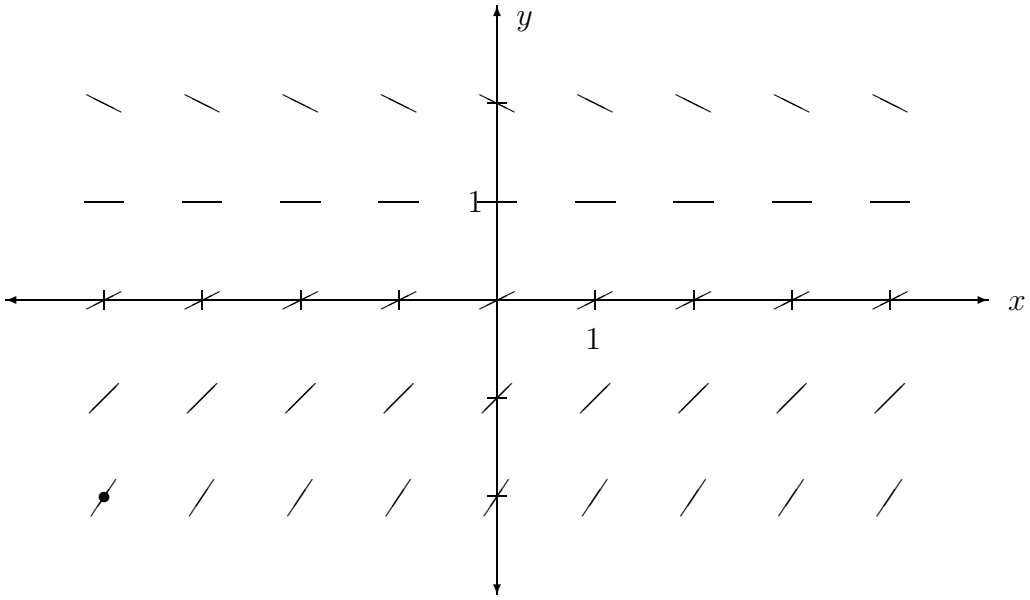
(b) Find the general solution to the differential equation $y' = \dfrac{\sec y}{1 + x^2}$. An implicit solution is enough here; you don't have to solve for y as a function of x.

11. (a) Find the general solution to the differential equation

$$(2x + y^3)\, dx + (y + 3xy^2)\, dy = 0.$$

An implicit solution is enough here; you don't need to solve for y in terms of x.

(b) The direction field for the differential equation $y' = (1 - y)/2$ is given below. Sketch the solution to this differential equation which passes through the point $(-4, -2)$.



12. Short answer. No explanations needed.

    (a) Does the existence and uniqueness theorem for solutions to IVPs that we discussed in class imply that the IVP $y' = xy^{1/3}$, $y(0) = 0$ has a unique solution?

    (b) Give an example of a second order *nonlinear* differential equation.

    (c) Is the ODE $-y^3\,dx + 3xy^2\,dy = 0$ exact?

    (d) How many solutions does the ODE $y' = x^3 + y^2$ have?

13. (a) Find the solution to the initial value problem $y' = 2(y+1)^{1/2}\cos x$, $y(0) = 3$.

    (b) Find the general solution to the differential equation $y' + 4y - e^{-x} = 0$.

14. (a) Find the solution to the initial value problem $y' + \dfrac{2y}{x-2} = \dfrac{1}{x}$, $y(1) = 4$.

    (b) Find the general solution to the differential equation $2xy\,dx + (y + x^2)\,dy = 0$. An implicit solution is enough here; you don't need to solve for y in terms of x.