Solution Quiz 1 (01/10/14)

① [#9, p.14]

assume $y = y(x)$ and

$$y - \ln y = x^2 + 1.$$

Then
$$\frac{d}{dx}\left(y - \ln y\right) = \frac{d}{dx}\left(x^2+1\right),$$

$$\frac{dy}{dx} - \frac{1}{y}\frac{dy}{dx} = 2x,$$

$$\frac{dy}{dx}\left(1 - \frac{1}{y}\right) = 2x,$$

$$\frac{dy}{dx} = \frac{2x}{1-\frac{1}{y}} = \frac{2xy}{\left(1-\frac{1}{y}\right)y} = \frac{2xy}{y-1}$$

assuming $y > 0$ & $y \neq 1$.

Thus the given relation is an implicit solution of the given differential equation.

(2) Show that for any constant c,

$$\phi(x) = x - 1 + ce^{-x}$$

is an explicit soln of the DE

$$\frac{dy}{dx} + y = x.$$

Suppose c is a constant and $y = x - 1 + ce^{-x}$.
Then

$$\frac{dy}{dx} = 1 + c(-1)e^{-x} = 1 - ce^{-x},$$

and

$$\frac{dy}{dx} + y = 1 - ce^{-x} + x - 1 + ce^{-x}$$

$$= x.$$

So $\phi(x)$ is a solution to the DE for any constant c.