(1) [2 points] Complete:

To solve the linear equation (*)

$$\frac{dy}{dx} + p(x)\, y = q(x) \qquad (*),$$

(i) we first let

$$\mu(x) = e^{P(x)},$$

where

$$P(x) = \underline{\int p(x)\, dx}.$$

(ii) Then we multiply both sides of (*) by $\underline{\mu(x)}$ and write the resulting equation as

$$\frac{d}{dx}\left(\underline{\mu(x)\, y}\right) = \underline{\mu(x)\, q(x)}.$$

(iii) The general solution of (*) is given by

$$y = \underline{\frac{1}{\mu(x)}\left(\int \mu(x) q(x)\, dx + C\right)},$$

where C is any constant.

(2) [8 points] Solve the IVP

$$x\frac{dy}{dx} + 4y = 1 - 7x^3, \quad y(1) = \frac{1}{4}.$$

$$x\frac{dy}{dx} + 4y = 1 - 7x^3$$

$$\Leftrightarrow \frac{dy}{dx} + \frac{4}{x}y = \frac{1}{x}(1-7x^3) \quad \text{for } x>0. \quad (*)$$

Let $P(x) = \int \frac{4}{x}\,dx = 4\ln x$ (for $x>0$) &

$\mu(x) = e^{P(x)} = e^{4\ln x} = x^4$. We multiply both sides of $(*)$ by x^4:

$$(*) \Leftrightarrow x^4\frac{dy}{dx} + 4x^3y = x^3(1-7x^3) \quad (\text{for } x>0)$$

$$\frac{d}{dx}\left(x^4y\right) = x^4\frac{dy}{dx} + 4x^3y$$

$$(*) \Leftrightarrow \frac{d}{dx}\left(x^4y\right) = x^3(1-7x^3)$$

$$x^4y = \int x^3(1-7x^3)\,dx = \int x^3 - 7x^6\,dx$$

$$(*) \Leftrightarrow x^4y = \frac{x^4}{4} - x^7 + c,$$ where c is constant.

$y(1) = \frac{1}{4}$: $\frac{1}{4} = \frac{1}{4} - 1 + c$ & $c = 1$.

Hence $x^4y = \frac{x^4}{4} - x^7 + 1$ &

$$y = \frac{1}{x^4}\left(\frac{x^4}{4} - x^7 + 1\right) \quad \text{for } x>0$$

is the explicit sol$\underline{\underline{n}}$.