MAP 2302 — QUIZ 5 — SPRING 2014

(1) [4 points] Complete:

Definition: A linear 2^{nd} order DE has the form
$a(t)y''+b(t)y'+c(t)y=f(t)$
where $a(t)$, $b(t)$, $c(t)$, $f(t)$ are continuous functions on an interval $I=(\alpha,\beta)$.

Theorem Let a, b, c be constants & suppose $a\neq 0$.
Suppose r is a real number that satisfies
$ar^2+br+c=0$ (Characteristic or Auxiliary Equation).
Then $y = e^{rt}$ is a solution of the homogeneous equation
$ay''+by'+cy=0$.

Theorem If y_1, y_2 are solutions of
(*) $ay''+by'+cy=0$,
then
$y = c_1y_1+c_2y_2$,
is also a solution where c_1, c_2 are any constants

Existence Uniqueness Theorem for 2^{nd} order linear IVPs with constant coefficients Let $t_0, a, b, c, Y_0, Y_1 \in \mathbb{R}$, $a\neq 0$.
The IVP
$ay''+by'+cy=0;\quad y(t_0)=Y_0,\ y'(t_0)=Y_1,$
has a unique solution valid on the interval $I=(-\infty,\infty)$.

Definition Two functions $y_1(t), y_2(t)$ (defined on an interval I) are linearly dependent on I if there are constants c_1, c_2 not both zero such that

$$c_1 y_1(t) + c_2 y_2(t) = 0$$

for all $t \in I$. Otherwise, they are linear independent on I.

Lemma Two functions $y_1(t), y_2(t)$ (on I) are linearly dependent on I if and only if at least one function is a constant multiple of the other.

(2) [6 points] Solve the IVP

$$y'' + y' - 2y = 0, \quad y(0) = 5, \quad y'(0) = -1.$$

AE: $r^2 + r - 2 = 0$

$(r+2)(r-1) = 0$

$r = 1, -2.$

$y_1 = e^t$, $y_2 = e^{-2t}$ are solutions.

Let $y = c_1 e^t + c_2 e^{-2t}$ where c_1, c_2 are constants.

$y(0) = c_1 + c_2 = 5.$

$y' = c_1 e^t - 2c_2 e^{-2t},$

$y'(0) = c_1 - 2c_2 = -1.$

$$\boxed{\begin{aligned} c_1 + c_2 &= 5 \\ c_1 - 2c_2 &= -1 \end{aligned}}$$

$3c_2 = 6$, $c_2 = 2$ & $c_1 = 3.$

Thus $y = 3e^t + 2e^{-2t}$ is the soln to the IVP.