MAP 2302 — QUIZ 6 — SPRING 2014 Solution

Question 1 [3 points] Complete:

Theorem (Characteristic Equation has complex roots)
Let a, b, c be constants, $a \neq 0$ and $\Delta = b^2 - 4ac$.
Suppose $\Delta < 0$. Then the auxiliary equation
$$ar^2 + br + c = 0$$
has two complex roots

$$r = \alpha \pm i\beta$$

where $\alpha = \frac{-b}{2a}$, $\beta = \frac{\sqrt{4ac - b^2}}{2a}$.

The differential equation
$$(*) \quad ay'' + by' + cy = 0$$
has two linearly independent solutions

$y_1 = e^{\alpha t} \cos \beta t$, $y_2 = e^{\alpha t} \sin \beta t$,

and the general solution of $(*)$ is given by
$$y = e^{\alpha t}(c_1 \cos \beta t + c_2 \sin \beta t),$$
where c_1, c_2 are any constants.

Question 2 [4 + 3 = 7 points]
(a) Find the general solution of the DE
$$y'' - 2y' + 5y = 0.$$

(b) Solve the IVP
$$y'' - 2y' + 5y = 0, \quad y(0) = 2, \quad y'(0) = 8.$$

2 (a) A.E. $r^2 - 2r + 5 = 0$

$$r^2 - 2r + 1 = -5 + 1$$

$$(r-1)^2 = -4$$

$$r - 1 = \pm 2i$$

$$r = 1 \pm 2i; \quad \alpha = 1, \beta = 2.$$

So $y_1 = e^t \cos 2t$, $y_2 = e^t \sin 2t$ are lin. indep. solns & the general soln is given by

$$y = e^t (c_1 \cos 2t + c_2 \sin 2t),$$

where c_1, c_2 are any constants.

(b) $y' = e^t(-c_1 \sin 2t + 2c_2 \cos 2t) + e^t(c_1 \cos 2t + c_2 \sin 2t)$.

$$y(0) = c_1 = 2.$$

$$y'(0) = 2c_2 + c_1 = 8, \quad 2c_2 = 6, \quad c_2 = 3.$$

The soln is

$$y = e^t (2 \cos 2t + 3 \sin 2t).$$