

MAP 2302 — QUIZ 7 — SPRING 2014

WEDNESDAY, APRIL 9

Name: Solution ($\bar{x} = 83.9\%$)

Instructions: All work should be written in a proper and coherent manner. Write in such a way that any student in the class can follow your work. When working problem show all your work. Answers with no work or explanation will receive no credit, unless otherwise specified. A Table of Laplace Transforms is supplied.

TOTAL POINTS: 10

PROBLEM:

Find the inverse Laplace transform $\mathcal{L}^{-1}\{F(s)\}$ where

$$F(s) = \frac{3s^2 - 8 + 3s}{(s-2)(s^2 + 2s + 2)}$$

Note that $s^2 + 2s + 2$ is irreducible since $\Delta = b^2 - 4c = 4 - 8 = -4 < 0$

+5
$$\frac{3s^2 - 8 + 3s}{(s-2)(s^2 + 2s + 2)} = \frac{A}{s-2} + \frac{Bs + C}{s^2 + 2s + 2} \quad (\text{Partial Fractions})$$

$$3s^2 - 8 + 3s = A(s^2 + 2s + 2) + (Bs + C)(s-2)$$

$s=2$: $12 - 8 + 6 = A(4 + 4 + 2)$
 $10 = 10A$ & $A = 1$.

$s=0$: $-8 = 2A - 2C = 2 - 2C$, $2C = 10$, $C = 5$.

Coeff s^2 : $3 = A + B$, $B = 3 - A = 3 - 1 = 2$.

$$\frac{3s^2 - 8 + 3s}{(s-2)(s^2 + 2s + 2)} = \frac{1}{s-2} + \frac{2s + 5}{(s+1)^2 + 1} = \frac{1}{s-2} + \frac{2(s+1)}{(s+1)^2 + 1} + \frac{3}{(s+1)^2 + 1}$$

+3
$$\mathcal{L}^{-1}\left\{\frac{3s^2 - 8 + 3s}{(s-2)(s^2 + 2s + 2)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} + \mathcal{L}^{-1}\left\{\frac{2(s+1)}{(s+1)^2 + 1}\right\} + 3 \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2 + 1}\right\}$$

+2
$$= e^{2t} + 2e^{-t} \cos t + 3e^{-t} \sin t$$