MAP 2302 — TEST 1 — SPRING 2008

Instructions: All work should be written in a proper and coherent manner. Write in such a way that any student in the class can follow your work. When working problems show all your work. Answers with no work or explanations will receive no credit, unless otherwise specified. A table of integrals is supplied. **ONLY SCIENTIFIC CALCULATORS ALLOWED**.

TOTAL POSSIBLE: 50 points.

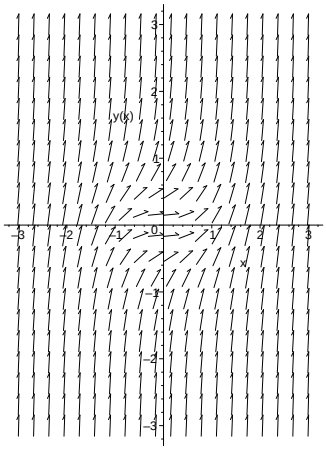
(1) [**2 points**] Given an example of an ordinary 2nd order linear differential equation.
(2) [**2 points**] Given an example of an ordinary 2nd order nonlinear differential equation.
(3) [**6 points**] Suppose the equation

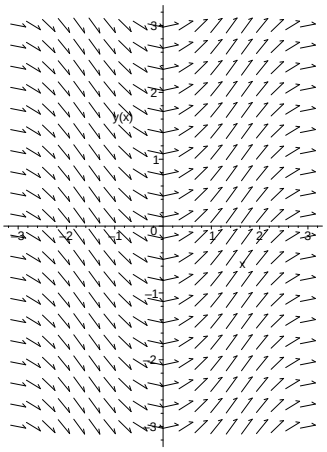
$$x^2 + \ell n(y) + \cos(x + y) = 1$$

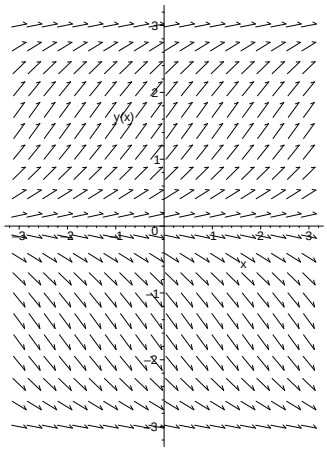
defines y implicitly as a function of x. By using implicit differentiation (or otherwise) find an ordinary first order differential equation satisfied by y.
(4) [**6 points**] Determine whether the Existence Uniqueness Theorem implies that the Initial Value Problem:

$$(x-1)y^3\frac{dy}{dx} = x + y^3, \qquad y(2) = 1,$$

has a unique solution on some interval containing $x = 2$. Show all reasoning.
(5) [**6 points**] Match the direction field plot with the differential equation. No working necessary.

(A) 

(B) 

(C) 

(i) $$\frac{dy}{dx} = \sin(x)$$

(ii) $$\frac{dy}{dx} = \sin(y)$$

(iii) $$\frac{dy}{dx} = \sin(x)\,\sin(y)$$

(iv) $$\frac{dy}{dx} = x^2 + 2y^2$$

(6) [**8 points**] Solve the differential equation

$$\frac{dy}{dx} = \frac{\csc^2 y}{1+x}.$$

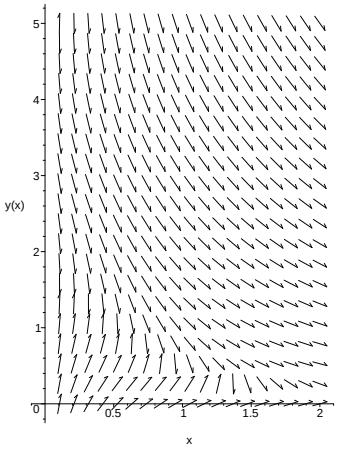
(7) [8 **points**] Solve the Initial Value Problem

$$x\,\frac{dy}{dx} + 4y = 1 - 7x^3, \qquad y(1) = \frac{1}{4}.$$

(8) [$10 + 2 = 12$ **points**] (i) Show that the Initial Value Problem

$$\left(\frac{1}{x} + 2y^2 x\right) + \left(2yx^2 - \cos(y)\right)\,\frac{dy}{dx} = 0, \quad y(1) = \pi.$$

(ii) Below is the direction field plot for the differential equation in part (i).



Plot the solution to the initial value problem (in (i)) on this direction field plot.