MAP 2302 — TEST 2 — SPRING 2008
NAME: _____ **PERIOD:** _____

Instructions: All work should be written in a proper and coherent manner. Write in such a way that any student in the class can follow your work. When working problems show all your work. Answers with no work or explanations will receive no credit, unless otherwise specified. A table of integrals is supplied. **ONLY SCIENTIFIC CALCULATORS ALLOWED**. **TOTAL POSSIBLE**: 53 points (50 points = 100%)

(1) [5 **points**] Let $y_1 = e^{5x}$. Find a second order linear differential operator

$$L[y] = L[y](x) = y''(x) + p_1(x)\, y'(x) + p_2(x)\, y(x)$$

such that
$$L[y_1] = 0.$$

(2) [**8 points**] Discuss what the Existence and Uniqueness Theorem for 2nd order linear differential equations implies about solutions to the initial value problem:

$$e^x y'' + \frac{1}{x-2} y' + xy = \ln x, \quad y(1) = 7, \quad y'(1) = -3.$$

(3) [$4 + 2 + 2 + 2 + 4 = 14$ **points**]
(a)   Verify that the functions

$$y_1 = e^{2x}$$
$$y_2 = 2x^2 + 2x + 1$$

are solutions to the differential equation

$$x\frac{d^2 y}{dx^2} - (2x+2)\frac{dy}{dx} + 4y = 0, \qquad x > 0.$$

(b)   Compute the Wronskian $W[y_1, y_2](x)$.
(c)   Explain why the solutions y_1, y_2 are linearly independent.
(d)   Find the general solution of this differential equation.
(e)   Find the solution that satisfies the initial conditions

$$y(1) = -10 + e^2,$$
$$y'(1) = -12 + 2e^2.$$

(4) [6 **points**] Find the general solution to the differential equation

$$y'' - 2y' + 2y = 0.$$

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(5) [10 **points**] Find the *form* of a particular solution $y_p(t)$ for the DE
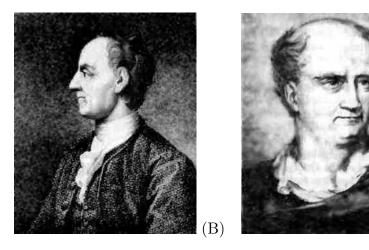
$$y'' - 2y' - 3y = t\,e^{3t} + t^2\cos 3t - 5\sin 3t.$$

Do NOT compute the coefficients but do explain how you obtained the form in detail.

(6) [**6 points**] Suppose $p(x)$ and $q(x)$ are continuous functions on the interval $(-1, 1)$. Determine whether the function $w(x) = 2x + 1$ could be the Wronksian of two solutions y_1 and y_2 of the differential equation

$$y''(x) + p(x)y'(x) + q(x)y(x) = 0,$$

for $-1 < x < 1$, giving reasons.

(7) [**4 points**] Name each guy and describe some of their mathematics.

(A)   (B)