Test 3 Review

Laplace Transform Let $f(t)$ be a function on ______. The Laplace transform of f is the function F defined by the integral

$$F(\_\_\_) := \_\_\_\_\_\_\_\_\_\_\_\_\_\_.$$

The domain of $F(\_\_\_)$ is all values of ____ for which the integral ________. The Laplace transform of f is denoted by both F and ________

Piecewise Continuity A function $f(t)$ is said to be piecewise continuous on a finite interval $[a,b]$ if $f(t)$ is ____________ at ______ point in $[a,b]$ except __________________ at which $f(t)$ has a _____________.
A function $f(t)$ is piecewise continuous on $[0,\infty)$ if $f(t)$ is ____________________________.

Exponential Order α
A function $f(t)$ is said to be of exponential order α if there exists positive constants T and M such that

____

Conditions for the Existence of the Transform
If $f(t)$ is __________________ on $[0,\infty)$ and of ____________________, then $\mathcal{L}\{f\}(s)$ exists for _______.

Inverse Laplace Transform

Given a function $F(s)$ if there is a function ___ that is ______ on $[0, \infty)$ and satisfies

$$\mathcal{L}\{\_\_\_\} = \_\_\_ ,$$

then we say ___ is the inverse Laplace transform of ___ and employ the notation

$$\_\_\_ = \mathcal{L}^{-1}\{\_\_\}.$$

Translation

Let a be a positive constant.

$$\mathcal{L}\{g(t)\,u(t-a)\}(s) = \_\_\_\_\_\_\_\_.$$

Taylor Polynomials

The n-th degree Taylor polynomial of $f(x)$ centered at x_0 is

$$p_n(x) = f(x_0) + \_\_\_(x-x_0) + \frac{\qquad}{\_\_\_}(x-x_0)^2$$

$$+ \cdots + \frac{\qquad}{\_\_\_}(x-x_0)^n$$

$$= \sum_{j=\_\_}^{n} \boxed{\qquad\qquad}$$

Power Series

A power series about the point x_0 is a expression of the form

$$\sum_{\_\_\_}^{\infty} \_\_\_ = a_0 + \_\_\_\_\_\_\_\_\_\_.$$

Analytic Function

A function f is analytic at x_0 if

$$f(x) = \sum_{n=0}^{\infty} \_\_\_\_\_\_ \quad \text{for } |x - x_0| < \_\_\_$$

for some ___

and the power series has positive radius convergence ___

ie the power series converges for ________.

Ordinary & Singular Points

A point x_0 is called an ordinary point of

$$y'' + p(x)y' + q(x)y = 0$$

if _____ $p(x)$ and $q(x)$ are ________.

If x_0 is not an ordinary point it is called a ________ point.

Existence of Analytic Solutions

Suppose x_0 is an ordinary point of

(*) $y'' + p(x)y' + q(x)y = 0$.

Then (*) has two linearly ___________ solutions of the form

(**) $y(x) =$ _________.

Moreover the radius of convergence of any power series solution of the form given by (*) is _________________

____

Differentiation of Power Series

Suppose $f(x) = \sum_{n=0}^{\infty} a_n (x - x_0)^n$

for $|x - x_0| < R$ where $R > 0$ & the series converges for these values of x. Then

$$f'(x) = \sum_{n=0}^{\infty} \_\_\_\_\_\_\_\_,$$

and

$$f''(x) = \sum \_\_\_\_\_\_\_\_$$

$$= \sum \_\_\_\_\_\_\_\_$$

for $|x - x_0| < \_\_\_\_.$