MAP 2302 — TEST 3 — SPRING 2008
NAME: _____ **PERIOD:** _____

Instructions: All work should be written in a proper and coherent manner. Write in such a way that any student in the class can follow your work. When working problems show all your work. Answers with no work or explanations will receive no credit, unless otherwise specified. Tables of integrals and Laplace transforms are supplied. **ONLY SCIENTIFIC CALCULATORS ALLOWED**. **TOTAL POSSIBLE**: 54 points (50 points = 100%)

(1) [6 + 2 + 2 = 10 **points**] A 1 kg mass is attached to a spring with stiffness 5 N/m. The damping constant is 4 N-sec/m. The mass is moved 3 m to the right of equilibrium and given a leftward velocity of 5 m/sec. Neglecting external forces that may be present find

(a)   The equation of motion

(b)   The damping factor

(c)   The quasi-period and quasi-frequency.

(2) [2 + 6 = 8 **points**]

(a)   Let $f(t)$ be a function defined on $[0, \infty)$. Define the **Laplace transform** of f.

(b)   Determine the Laplace transform $\mathcal{L}\{1\}$ from first principles; ie. using only the definition of the Laplace transform.

(3) [2 + 3 + 3 = 8 **points**]
Determine the Laplace transform of the given function using the table and properties of the Laplace transform.

(a) $3t^4 + e^{3t} \sin 2t - t \cos t$

(b) $\sin(2t) \cos(3t)$

(c) $t^2 \sin t$

(4) [6 **points**] For the initial value problem

$$y'' + 5y' + 6y = e^{2t} \cos t + \sin 2t, \qquad y(0) = 1, \; y'(0) = -2,$$

find $Y(s)$ the Laplace transform of the solution $y(t)$.

Do NOT find the solution $y(t)$.

(5) [4 **points**]

Express $f(t)$ (see graph above) in terms of the unit step function.

(6) $[2 + 6 = 8$ **points**] Let

$$f(t) = t^2(u(t-1) - u(t-2))$$

(a) Sketch the graph of $f(t)$.

(b) Find the Laplace transform of $f(t)$.

(7) [6 **points**] Find
$$\mathcal{L}^{-1}\left\{\frac{e^{-3s}(2s^2 + 4s + 3)}{(s+1)^2(s+2)}\right\}.$$

(8) [4 **points**] Name each guy. For each person, describe some aspect of their life OR their mathematics. Write in complete sentences.

(A)   (B)   (C)