MAS 4203 Exam

Start 1:55 pm

Finish 3:20 pm

Inst. F. GARVAN
LIT 408 (Math)
870-3886
fgarvan@ufl.edu

Extra paper allowed.
Only basic or scientific calculators are allowed.

MAS 4203 - EXAM - Summer 2015

Thursday, July 16

NAME:

Instructions: All work should be written in a proper and coherent manner, and in a way that any student in the class can follow your work. Show all necessary working and reasoning. When giving proofs your reasoning should be clear. Only scientific or basic calculators are allowed.

TOTAL

1. [$2 + 4\times 2 = 10$ pts]

(a) Complete the definition: Let $a, b \in \mathbb{Z}$. Then a divides b denoted _______, if ______________________.

(b) Prove or disprove the following statements:

(i) If $a \in \mathbb{Z}$ and $a \mid 0$ then $a = 0$.

(ii) There are integers x, y such that
$$3x - 453y = 347.$$

(iii) If $a, b, c \in \mathbb{Z}$ and $a \mid bc$ then $a \mid b$ or $a \mid c$.

(iv) If $a, b, c, d \in \mathbb{Z}$, $a \mid b$ and $c \mid d$ then $ac \mid bd$.

2. [2 + (2+2) + 2 + 2 = 10 pts]

(a) Complete the following

<u>Theorem</u> : Let $a = p_1^{e_1} p_2^{e_2} \cdots p_r^{e_r}$,

$b = p_1^{f_1} p_2^{f_2} \cdots p_r^{f_r}$

be prime factorizations with each $a_i, b_j \geq 0$.

Then $a \mid b$ if and only if ________________.

(b) Prove or disprove the following statements

(i) If $a, b \in \mathbb{Z}$ $a, b > 0$ and $a^3 \mid b^4$ then $a \mid b$.

(ii) If $a \in \mathbb{Z}$, $a > 0$, p is prime & $p^5 \mid a^2$ then $p^3 \mid a$.

(c) Complete the Proposition: Let $a, b \in \mathbb{Z}$ with __________.
Then
$(a,b) =$ ___ $\{$___ : __________$\}$

(d) PROVE OR DISPROVE
If a, b, c are positive integers such that $(a,c) = (b,c) = 1$, then for any positive integers m and n
$$(am + bn, c) = 1.$$

3. [2+3+5 = 10 pts]
(a) Complete the Definition: Let $p \in \mathbb{Z}$ and _____.
Then p is said to be prime if __________
____________________.

(b) Let $a, b \in \mathbb{Z}$. Prove that if a and b are expressible in the form $6n+1$ where n is an integer, then ab is also expressible in that form.

(c) Prove that there are infinitely many primes of the form $6n+5$ where n is an integer as follows: Suppose by way of contradiction that there are only finitely many primes of the form $6n+5$ say

$p_0 = 5, p_1, p_2, \ldots, p_k$.

Let

$$N = 6(\_\_\_\_\_\_\_\_\_\_) + (\_\_\_).$$

4. [2 + 4 + 2 + 2 = 10 pts]

(a) Complete the Definition: Let $a, b, m \in \mathbb{Z}$ with $m \geq 1$. Then a is said to be congruent to b modulo m, denoted ________, if ____________________.

(b) PROVE: If $a, b, c, d \in \mathbb{Z}$ and $a \equiv b \pmod{m}$, and $c \equiv d \pmod{m}$ then $a + c \equiv b + c \pmod{m}$.

(c) Complete the Definition: Let $m \in \mathbb{Z}$, $m \geq 1$. A complete residue system modulo m is a ____________________

____

________________.

(d) Prove or disprove that set $\{0^2, 1^2, 2^2, 3^2, 4^2, 5^2, 6^2\}$ is a complete residue system mod 7.

5. [2 + 5 + 3 = 10 pts]

(a) Complete

<u>Fermat's Little Theorem</u> Let $a \in \mathbb{Z}$, p be ____ and suppose ______. Then

$$a^{---} \equiv \_\_\_ \pmod{\_\_\_}.$$

(b) Prove

<u>Corollary to Fermat's Little Theorem</u>

Let $a \in \mathbb{Z}$ and p be prime. Then

$$a^p \equiv a \pmod{p}.$$

<u>PROOF:</u>

(c) Suppose $x \in \mathbb{Z}$, $x \not\equiv 0 \pmod 5$, and $x \not\equiv 1 \pmod 5$. Prove that

$$x^3 + x^2 + x + 1 \equiv 0 \pmod 5.$$

(d) [BONUS 5 pts]
Generalize the result of (c) to any odd prime p and prove your result.

6. [2+8 = 10 pts]

(a) Complete the Definition. Let n be a ________ integer. If ________, then n is said to be pseudoprime.

(b) Prove only ONE part:

(i) $645 = (3)(5)(43)$ is pseudoprime.

(ii) If p is prime and $n = 2^p - 1$ is composite then n is pseudoprime.

(iii) Let $a \in \mathbb{Z}$ and suppose p and q are distinct primes. Then

$$a^{pq} + a \equiv a^p + a^q \pmod{pq}.$$

7. [3 bonus points]

(a) This is _ _ _ _ _ _ _ _

C_ _ _ F_ _ _ _ _ _ _ _

_ _ _ _ _ _

(b)

D_ _ _ _ _ _ _ _ _ _ _ _ _ _es

A_ _ _ _ _ _ _ _ _ _ _ae.

is the title of his famous book on Number Theory

(c) He left G_ _ _ _ _ _ _en in 1798 without a diploma, but by this time he had made one of his most important discoveries — the construction of the regular _ _-gon by ruler and compasses.