MAS 4203 – QUIZ 1 – SUMMER 2014

~~Wednesday, July 9~~ **MAKE-UP**

NAME:

Instructions: All work should be written in a proper and coherent manner, and in a way that any student in the class can follow your work. Show all necessary working and reasoning. When giving proofs your reasoning should be clear. Only scientific or basic calculators are allowed.

TOTAL: 15 + 3 bonus pts.

1. [1 + (1 + 2 + 1) = 5 pts]

(a) Complete the Definition: Let $a, b \in \mathbb{Z}$. Then a divides b denoted ______, if ________________________.

(b) Prove or disprove the following statements.

(i) If a, b, c, and d are integers such that $a \mid b$ and $c \mid d$, then $a+c \mid b+d$.

(ii) If a, b, c, and d are integers such that $a \mid b$ and $c \mid d$ then $ac \mid bd$.

(iii) If a, b, and c are integers such that $a \nmid b$ and $b \nmid c$ then $a \nmid c$.

2. [1+3+1 = 5 pts]

(a) Complete the Definition: Let $p \in \mathbb{Z}$ and ______. Then p is said to be *prime* if ______________________________

(b) Prove that ~~there are infinitely many primes.~~
n has a prime divisor $p \leq \sqrt{n}$ if n is composit

(c) Complete ~~Goldbach's Conjecture: Every even integer~~ Twin Prime Conjecture: There are infinitely ~~... than~~ many. ______________________________

3. [1+1+3 = 5 pts]

(a) Complete the definition: Let $a, b \in \mathbb{Z}$ with ______________________________. The greatest common divisor of a and b, denoted ______, is the

(b) Complete the Proposition: Let $a, b \in \mathbb{Z}$ with ______________________________. Then
$(a, b) =$ ____ $\{$ ________ $:$ ______________________ $\}$.

(c) Use the Proposition in (b) (and not the Fundamental Theorem of Arithmetic) to prove the following

Theorem Let $a, b, c \in \mathbb{Z}$ with $(a, b) = 1$.
If ~~a | bc then a | c.~~
$a | c$ and $b | c$ then $ab | c$.

PROOF.

4. [BONUS]. [1+1+1= 3 bonus pts]

(a) Who is this guy?

(b) What did he think of Galileo?

(c) Which curve did he study which he referred to as the 'roulette'?