MAC 4203 - Quiz 3 - Summer 2014 MAKE-UP
[redacted]

Name:

Instructions: All work should be written in a proper and coherent manner, and in a way that any student in the class can follow your work. Show all necessary working and reasoning. When giving proofs your reasoning should be clear. Only scientific or basic calculators are allowed.

Total: 15 + 3 bonus points

1. [1+1+3]

(a) Complete the definition: Let f be an arithmetic function. Then f is multiplicative if

(b) Complete the theorem: Let f be an arithmetic function and, for $n \in \mathbf{Z}$ with $n > 0$, let

$$F(n) = \sum_{d|n, d>0} f(d).$$

If f is ______________, then F is ______________.

(c) [redacted]

Prove that there are infinitely many integers n for which $\phi(n) = \frac{n}{3}$.

2.[1+2+2]

$\nu(15360)$

a. Compute [redacted] ($15360 = 2^{10} \cdot 3 \cdot 5$) where $\nu(n)$ is the number of positive divisors of n.

b. Find the least nonnegative residue [redacted] modulo 20 of 29^{198} & find $\varphi(198)$.

c. Prove if $n \in \mathbf{Z}$ and $n > 0$ then [redacted] $n^7 \equiv n^{227} \pmod{11}$.

3.[2+1+2] Recall for $n \in \mathbf{Z}$ and $n > 0$, $\nu(n)$ is the number of positive divisors of n.

a. Prove or disprove: ν is multiplicative.

b. Prove or disprove: ν is completely multiplicative.

c. Prove ~~if $p_1, p_2, \ldots, p_m$ are distinct primes, then $\nu(p_1 p_2 \ldots p_m) = 2^m$.~~

that $\nu(n)$ is odd if and only if n is a perfect square.

4. [BONUS] [1+1+1]

a. Who is this guy?

b. Give the statements of two theorems named after him.

c. What did Descartes think of him?