

MAS 4203 - Quiz 4 - Summer 2014

~~Wednesday, July 30~~

MAKE-UP

NAME: _____

Instructions: all work should be written in a proper and coherent manner, and in a way that any student in the class can follow your work. Show all necessary reasoning and working. When giving proofs your reasoning should be clear. Only scientific or basic calculators are allowed.

TOTAL: _____

1. [1+1+3 = 5pts]

(a) Complete the Definition: Let $n \in \mathbb{Z}$ with $n > 0$.

The sum of positive divisors function denoted _____ is the function defined by

$$\sigma(n) = \text{-----}$$

(b) Complete the Theorem. Let p be a prime number

and let $a \in \mathbb{Z}$ with $a > 0$. Then

$$\sigma(p^a) = \text{-----}$$

(c) Let $n \in \mathbb{Z}$ with $n > 0$. Prove that

~~MAJOR~~

$$n \leq \sigma(n) \leq n^2$$

2. [1+1+3 = 5pts]

(a) Complete the Definition: Let $n \in \mathbb{Z}$ with $n > 0$.

The Möbius μ -function denoted $\mu(n)$ is

$$\mu(n) = \begin{cases} 1 & \text{if } n=1, \\ (-1)^k & \text{if } p^2 \nmid n \text{ with } p \text{ prime,} \\ 0 & \text{if } n = \dots \\ & \text{with } \dots \end{cases}$$

(b) Complete the Theorem. Let $n \in \mathbb{Z}$ with $n > 0$.

Then

$$\sum_{d|n} \mu(d) = \begin{cases} 1 & \text{if } n=1 \\ 0 & \text{if } n > 1. \end{cases}$$

(c) ~~Let $F(n) = \sum_{d|n} |\mu(d)|$.~~

~~(i) Explain why $F(n)$ is multiplicative.~~

Define $\Lambda(n) = \begin{cases} \ln p & \text{if } n=p, p \text{ prime} \\ 0 & \text{otherwise} \end{cases}$

Prove

$$\Lambda(n) = - \sum_{d|n} \mu(d) \ln(d)$$

3. [2 + 3 = 5 pts]

3. Complete the Definition:

(a) Let $a, m \in \mathbb{Z}$ with $m > 0$ and $(a, m) = 1$.

Then a is said to be a quadratic residue modulo m if _____

(b) Find the quadratic residues mod ~~11~~ 11.

(4)

4. [1+1+1 = 3 bonus pts]



(a) Who is this guy?

(b) What did he think
of Gauss?

(c) In three volumes in
1811, 1817 and 1819

he published work

on _____

functions.