MAS 4203 - EXAM 2 - Spring 2016

Friday, March 18

NAME:

Instructions. All work should be written in a proper and coherent manner, and in a way any student in the class can follow your work. Show all reasoning and working. When giving proofs make sure your reasoning is clear. Only scientific or basic calculators are allowed.

THERE ARE SIX QUESTIONS OF EQUAL VALUE.

DO ONLY FIVE COMPLETE QUESTIONS.

1. [10] Let $n \in \mathbb{Z}$ with $n > 1$. PROVE: If $(n-1)! \equiv -1 \pmod{n}$ then n is prime.

2. [2+8=10] (i) Define the term pseudoprime

(ii) PROVE that $561 = (3)(11)(17)$ is pseudoprime.

3. [2+8=10] (i) State Euler's Theorem.

(ii) Let m, n be positive relatively prime integers. PROVE

$$m^{\phi(n)} + n^{\phi(m)} \equiv 1 \pmod{mn}.$$

4. [2+8=10] (i) Define multiplicative.

(ii) PROVE that the function

$$f(n) = \begin{cases} -1 & \text{if } n \text{ is even} \\ 1 & \text{if } n \text{ is odd} \end{cases}$$

is multiplicative.

5. [$5 \times 2 = 10$] Find

(i) $\phi(3000)$

(ii) $\tau(3000)$

(iii) $\sigma(30)$

(iv) $\mu(30)$

(v) $F(175)$ if $F(n) = \sum_{d|n} (\tau(d))^2$.

6. [$3 + 4 + 3 = 10$].

(i) Find $g(25)$ if $(\sigma(n))^2 = \sum_{d|n} g(d)$.

(ii) PROVE that there are infinitely many positive integers n such that $\mu(n) + \mu(n+1) = 0$. [Hint: When is $\mu(n) = 0$?]

(iii) n is abundant if $\sigma(n) > 2n$. PROVE: If $2^n - 1$ is composite then $2^{n-1}(2^n - 1)$ is abundant.

BONUS

7. [3 bonus points]

(a) This is

C___ F___

(b)

D___es

A___ae

is the title of his famous book on Number Theory

(c) He left G___en in 1798 without a diploma, but by this time he had made one of his most important discoveries — the construction of the regular __-gon by ruler and compasses.