Homework 7/6

In each problem, use the limit definition of the slope of the tangent line given in class:

\[
\lim_{x \to a} \frac{f(x) - f(a)}{x - a} \quad \text{or} \quad \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}
\]

1. Find the slope of the tangent line to \( f(x) = \frac{x}{x+1} \) at \( x = 2 \).

2. Find the equation of the tangent line to \( y = \sqrt{x} - 1 \) at \( x = 4 \).

3. Show that for the line \( y = mx + b \), the slope of the tangent line at any point \( a \) equals \( m \) (the slope of the line).

4. Suppose the height of a ball thrown straight up into the air from the ground is given by \( h(t) = 40t - 16t^2 \).
   
   a) Find the average velocity of the ball on the intervals \([2, 3], [2, 2.5], [2, 2.1]\).
   
   b) Find the instantaneous velocity of the ball at \( t = 2 \).
   
   c) Find the time when the ball lands on the ground again after being thrown, and find the instantaneous velocity of the ball at this time.