1. Suppose $A, B$ are sets and that $A$ is finite. Assume $A \sim B$. Show that $B$ is also finite and that $|A| = |B|$.

2. Let $B$ be a set, and let $A = B \cup \{x\}$ where $x \notin B$. Prove that

$$\mathcal{P}(A) = \mathcal{P}(B) \cup \left\{ \{x\} \cup C : C \in \mathcal{P}(B) \right\}.$$  

3. Show that if $A$ is finite, then $\mathcal{P}(A)$ is finite and $|\mathcal{P}(A)| = 2^{|A|}$. (Use problem number 2, and induction on the cardinality of $A$.)

4. Let $A_1, \ldots, A_n$ be pairwise disjoint sets, i.e. for all $i, j \leq n$ if $i \neq j$ then $A_i \cap A_j = \emptyset$. Prove (by induction on $n \geq 2$) that

$$\left| \bigcup_{i=1}^{n} A_i \right| = \sum_{i=1}^{n} |A_i|.$$  

5. Show that if $A_1, \ldots, A_n$ are finite sets, then $\bigcup_{i=1}^{n} A_i$ is finite. (Reduce to the pairwise disjoint case and use problem 4.)

6. Prove that if $A, B$ are finite sets and if $f : A \rightarrow B$ is injective, then $|A| \leq |B|$.

7. Suppose $x, y, z \in \mathbb{Z}$ are distinct. Prove that it must be possible to choose a pair whose sum is even. (Hint: Pigeonhole Principle.)

8. Suppose there is a bank where 15,000 people have debit cards. Prove that there must be at least two people with the same PIN number. (Remember that a PIN number is 4 digits long, using digits 0-9.)