Final Exam Practice Problems.

1. Suppose $A$ is infinite, $B$ is finite, and $f : A \to B$. Prove that there is a $b \in B$ so $f^{-1}(b) = \{x \in A : f(x) = b\}$ is infinite.

2. Prove that $\mathcal{P}(A)$ is infinite iff $A$ is infinite.

3. Prove if $A$ is uncountable, then $\mathcal{P}(A)$ is uncountable. What about the converse?

4. Prove that for any set $A$, $\mathcal{P}(A)$ is not denumerable.

5. Show $(-\infty, 1) \sim [2, \infty)$.

6. Prove $[0, 1) \sim \mathbb{R}$.

7. Suppose there are injections $f : A \to B$, $g : B \to C$, and $h : C \to A$. Show that $A, B, C$ are all equinumerous.

8. Use a diagonal argument to show that the set of all functions $f : \mathbb{N} \to \mathbb{N}$ is uncountable.