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THE APPLICATION OF EIGENPAIR STABILITY TO BLOCK DIAGONALIZATION

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Abstract. An algorithm presented in Hager [Comput. Math. Appl., 14 (1987), pp. 561–572] for diagonalizing a matrix is generalized to a block matrix setting. It is shown that the resulting algorithm is locally quadratically convergent. A global convergence proof is given for matrices with separated eigenvalues and with relatively small off-diagonal elements. Numerical examples along with comparisons to the QR method are presented.

Key words. block diagonalization, eigenpair stability, eigenvalues, eigenvectors

AMS subject classifications. 65F05, 65F15, 65F35

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1. Introduction. Let A denote an $n \times n$ block matrix; that is, the (i, j) element A_{ij} of A is itself a matrix of dimension $n_i \times n_j$, where $\sum_{i=1}^{r} n_i = n$ for some $r \le n$. In this paper, we develop and analyze an algorithm for computing a block diagonalization $X\Lambda X^{-1}$ of A, assuming one exists. Here Λ is a block diagonal matrix with diagonal blocks Λ_i, $i = 1$ to r, and X is an invertible matrix whose ith block of columns is denoted by X_i. Hence, the equation $A = X\Lambda X^{-1}$ is equivalent to the relation

$$AX_i = X_i\Lambda_i, \qquad i = 1 \text{ to } r.$$

The algorithm developed in this paper is based on a stability result, Proposition 1, for a perturbation $A(\epsilon)X(\epsilon) = X(\epsilon)\Lambda(\epsilon)$ of the original eigenequation. We show that if the spectrum of Λ_i and Λ_j are disjoint for each $i \ne j$, then there exist continuously differentiable solutions $X(\epsilon)$ and $\Lambda(\epsilon)$ to the perturbed equation. After differentiating the perturbed equation and applying Taylor's theorem, we obtain the following algorithm (throughout the paper, the subscript k denotes the iteration number while the subscripts i and j denote elements or submatrices of larger matrices).

BLOCK DIAGONALIZATION ALGORITHM. If $X_k\Lambda_k X_k^{-1}$ is the current approximate diagonalization of A, then

$$\Lambda_{k+1} = \operatorname{diag} X_k^{-1}AX_k \quad \text{and} \quad X_{k+1} = X_k(I + D), \tag{1}$$

where

$$\operatorname{diag} D = 0 \quad \text{and} \quad D\Lambda_{k+1} - \Lambda_{k+1}D = \operatorname{off} X_k^{-1}AX_k. \tag{2}$$

The notation "diag" and "off" above are defined in the following way: given a block matrix B, "diag B" denotes the block diagonal matrix whose diagonal blocks coincide with the diagonal blocks of B, and "off B" denotes the matrix that coincides with B except for the diagonal blocks which are replaced by blocks of zeros.

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