Electric Field Reversal in Sprite Electric Field Signature

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ABSTRACT

In measurements of the electric field associated with the current of a sprite 450 km from ground-based field sensors, it was observed that the sign of the electric field was positive when positive charge was lowered from the ionosphere. A recent model for the electric field associated with the sprite current also predicts positive field-changes at 450 km from the sprite. A well-known analysis of a vertical dipole in a thundercloud shows that the electric field on the ground reverses its sign at an easily computed distance from the dipole. A similar simplified electrostatic analysis of a sprite predicts a field reversal distance around 130 km. A more accurate electrodynamic analysis based on Maxwell's equations indicates that the field reversal distance should be between 70 and 80 km.

1. Introduction

In a recent paper (Hager et al. 2012), we studied a series of sprites, luminous glows in the mesosphere above thunderclouds extending roughly from 50 to 90 km in altitude. Figure 1 shows a picture of a carrot sprite taken from Langmuir Laboratory, about 467 km west of the sprite, on 15 July 2010 at 05:27:09.69 UT. Three instruments (slow-antennae) in the Langmuir Electric Field Array (LEFA) measured the electric field during the storm. Figure 2 shows the vertical electric field that was measured at LEFA Station #2 during the carrot sprite. Observe that the electric field is predominantly positive while the sprite is descending from the ionosphere. A positive hump in the electric field from a sprite was also reported by Stanley et al. (2000) where it is referred to as the sprite's signature. In this paper, we study the dependence of the electric field on the distance from the sprite to the observer. We show that for the carrot sprite of Figure 1, the sign of the electric field should change from positive to negative as the distance to the sprite decreases. For this particular sprite, the field reversal distance is between 70 and 80 km.

2. Simplified Electrostatic Model

Before launching into a more complete model for the electric field from a sprite, let us first develop our intuition by reviewing a well-known analysis of a thunderstorm electric field (recently re-published by Rakov and Uman (2003)). Figure 3 shows a vertical dipole charge over a perfectly conducting plane with a positive charge at height h_p underneath an equal negative charge at height h_n. (In analyzing a storm, one usually puts the positive charge over the negative charge, but we have a reason for switching the charges in this discussion.) When the observer on the ground is at a location P_1 close to the dipole, the lower positive charge results in a negative electric field (pointing downward). When the observer is at a location P_2 far from the dipole, both charges are about the same distance from the observer and produce fields of roughly the same magnitude. However, the negative charge at higher altitude has a larger vertical component (by simple trigonometry), resulting in a net positive electric field at the observer. The intermediate point between P_1 and P_2 where the electric field vanishes yields the field-reversal distance D_0 given by the formula

$$D_0 = \sqrt{(h_p h_n)^\alpha (h_p^\alpha + h_n^\alpha)}, \quad \alpha = 2/3. \tag{1}$$

It is thought that a sprite is a manifestation of classical breakdown caused by the increased fields above a storm that has just experienced a large positive cloud-to-ground flash; for example, see the theory of Pasko et al. (1997). In a grossly simplified model of a sprite, we can consider it as inserting a positive charge in the atmosphere descending from the ionosphere. On time scales at which the ionosphere can be modeled as a perfect conductor, the positive charge which is the sprite leader tip should be mirrored be an equal and opposite negative charge which ascends above the ionosphere. Thus, figure 3, which at first glance appears to be a model of a thundercloud (with polarity reversed from the typical case), can be considered, with the addition of a conducting ionosphere midway between the two charges, to be an electrostatic model of a sprite.

If the simplified figure is the same, then the simplified math is also the same, and we can apply equation (1) to sprites. If the ionosphere is located at 100 km and the height of the positive charge is $h_p = 50$ km, then the height of the height of the negative image charge is $h_n = 150$ km. and the field reversal distance D_0 is about 127 km.

This calculation oversimplified the true physics. Both the conductive properties of the ionosphere and the conductivity of the surface of the earth must be accounted for. Moreover, for distant electromagnetic disturbances, the electrostatic contribution to the electric field to which equation 1 applies is often much smaller than the inductive and radiation contributions to the field. The next section of this paper provides a more accurate model for the electric field associated with the sprite current.

3. Modeled Electric Field for Sprite

In (Hager et al. 2012) a model for the electric field from a sprite is developed and is based on the following approximations: The earth and ionosphere are treated as perfectly conducting horizontal planes, and the sprite current is assumed to traveling along an infinitely thin wire connecting the altitude z_0 and the ionosphere at altitude H. The formula for a special exact solution to Maxwell's equation given by Uman et al. (1975), leads to the following relation for the vertical electric field at an observation point P on the ground:

$$E(t) = \sum_{k=1}^{\infty} E_k(t). \tag{2}$$

where

$$E_k(t) = \frac{(-1)^{k+1}}{2\pi\varepsilon_0}\left[\int_{z_0}^{H}\int_0^t \left(\frac{2-3\sin^2\theta_k(z)}{R_k(z)^3}\right) i\left(z,\tau-\frac{R(z)}{c}\right)d\tau dz\right.$$
$$\left. + \int_{z_0}^{H}\left(\frac{2-3\sin^2\theta_k(z)}{cR_k(z)^2}\right) i\left(z,t-\frac{R(z)}{c}\right)dz - \int_{z_0}^{H}\frac{\sin^2\theta_k(z)}{c^2R_k(z)}\frac{\partial i(z,t-R(z)/c)}{\partial t}dz\right]. \tag{3}$$

Here i is the current in the sprite, and if D denotes the distance from P to the base of the sprite, then $R(z) = \sqrt{D^2 + z^2}$. Thus $R(z)$ is the distance between a point on the sprite at altitude z and the observer. We also define $R_k(z) = \sqrt{D^2 + z_k^2}$ where

$$z_k = \begin{cases} kH - z & \text{if } k \text{ is even,} \\ kH + z - H & \text{if } k \text{ is odd,} \end{cases}$$

and $\sin\theta_k(z) = D/R_k(z)$. The three terms on the right side of equation (3) are often called the electrostatic term, the induction term, and the radiation term.

The formula (2)–(3) was derived using image charge techniques for a *dipole current generator*. The parameters $R_k(z)$ give the location relative to the observation point P of the image dipole current generators associated with the source generator at altitude z (see Figure 4). The current i was modeled as in a transmission line (see (Uman and McLain 1969)):

$$i(z,t) = i(t + z/v),$$

where z is altitude and v is the velocity of the downward descending current pulse. If we take $v = 0.4c$, which approximates the mean velocity of about 0.37c for a lighting return stroke reported by Idone and Orville (1982), then the sprite current that best fits the measured electric field at LEFA #2 is shown in Figure 5. The current is predominantly negative which indicates that positive charge is transported down from ionosphere by the sprite tip.

4. Field Reversal Distance and Discussion

The estimated sprite current, shown in Figure 5, can now be inserted in (2)–(3) to obtain the electric field at various distances from the sprite. In Figure 6, the electric field at distances from 60 km to 90 km is shown (For comparison, the modeled and measured field at 442 km is also shown). Observe that between 70 km and 80 km, the field changes from negative, to almost zero, to positive. The simplified electrostatic model is useful for giving intuition about what one expects, and it gives a field reversal distance within a factor of 2 of the more accurate model based on an exact solution of Maxwell's equations and an infinite number of image dipoles.

Note that the E-field curve at 442 km returns to zero at the end of the plot while the nearer models do not return to zero. This is to be expected. The electrostatic term of the exact solution depends on the cube of distance from the sprite, while the inductive and radiation terms have an $1/R^2$ and $1/R$ dependence respectively. Thus at the larger distance the field returns to zero because the current has gone to zero. However, at closer distances, the electrostatic term keeps the field away from zero because there is now a net charge that has moved as the result of the sprite. The field reversal distance for a sprite is of course much larger than field reversal distances often observed for lightning in thunderstorms. For example, if $h_p = 10$ km and $h_n = 5$ km, typical values for a thunderstorm, then the field reversal distance is about 10 km, which can be compared to an estimated field reversal distance between 70 and 80 km for the sprite of 15 July 2010. To our knowledge, no one has reported observing sprite-electric fields from within the reversal distance, but we hope this happens soon.

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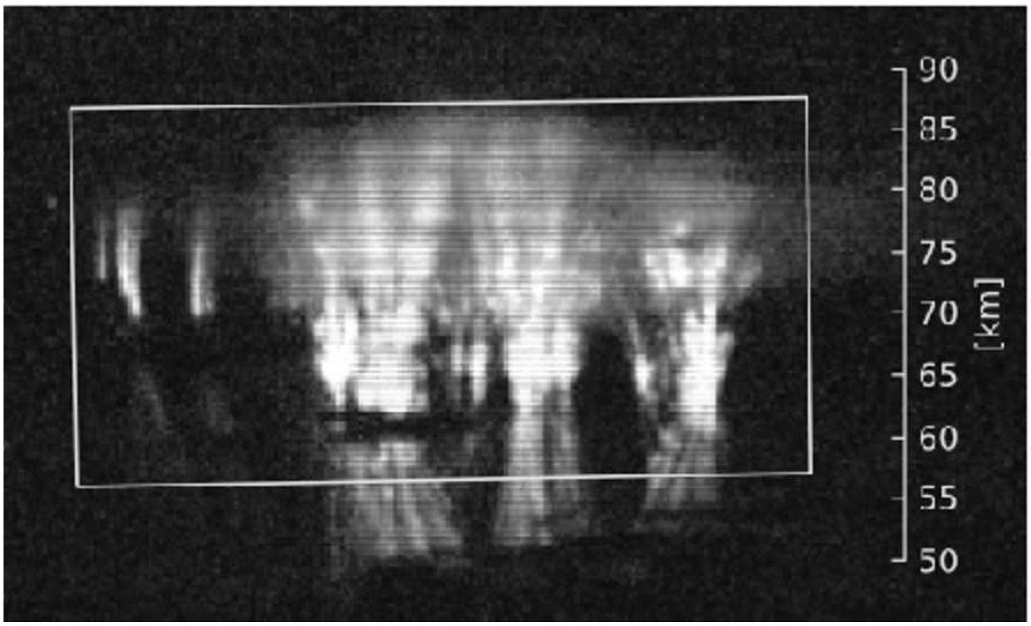

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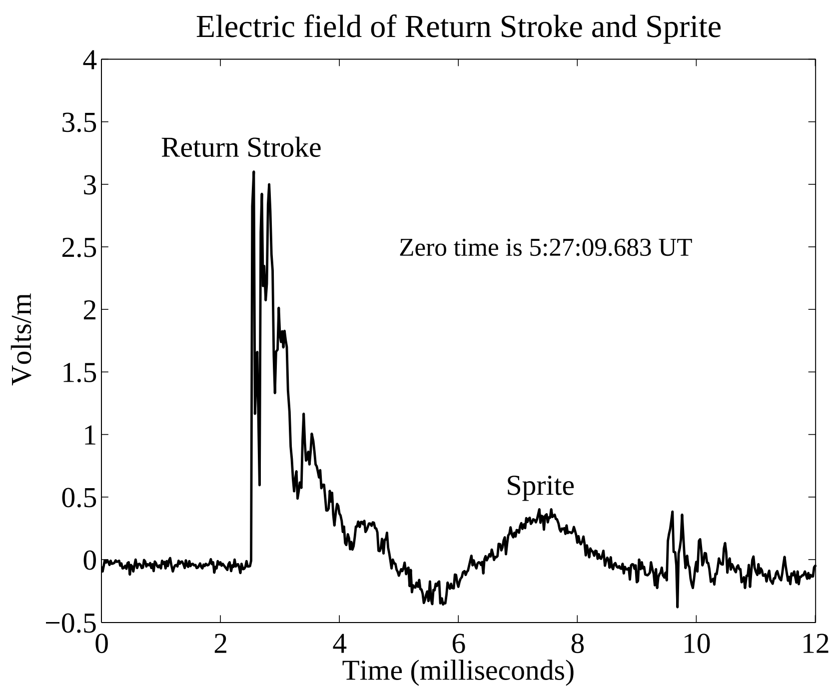
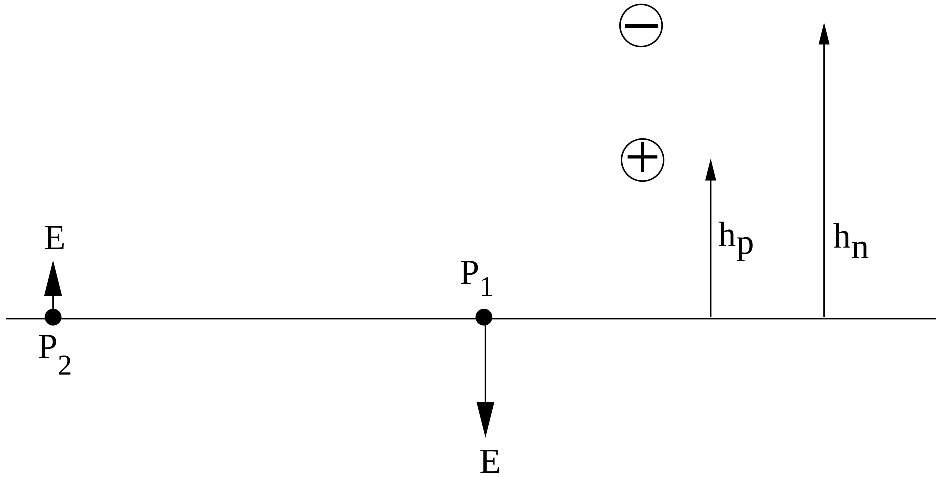


Fig. 2. Electric field seen at a point 450 km West of a large + cloud-to-ground flash. The hump labeled "sprite" coincides to within a fraction of a millisecond with the peak in a light curve of the sprite produced by that flash.



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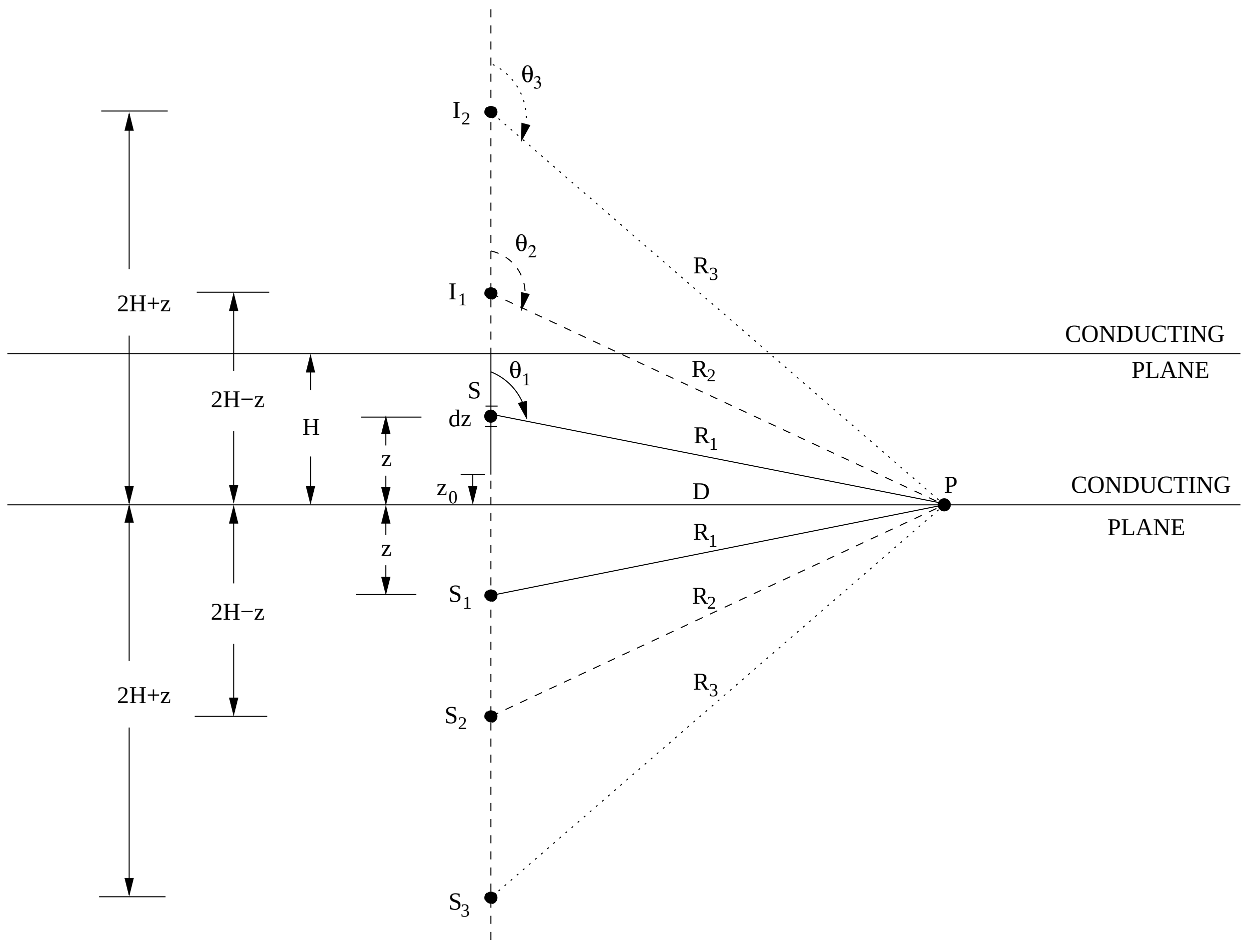

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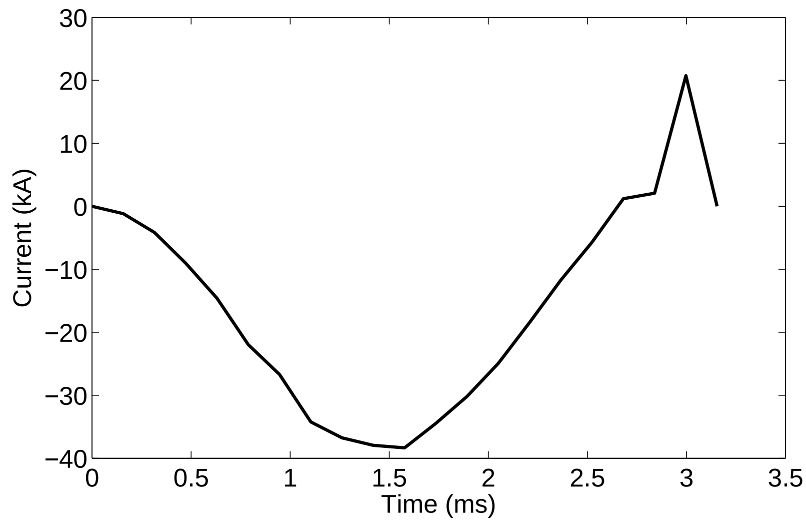
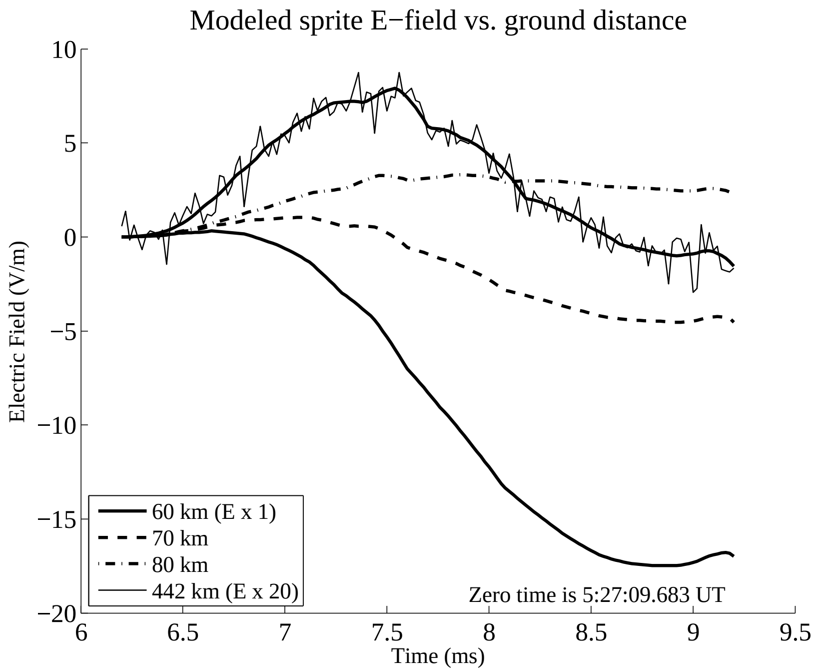


Fig. 5. The sprite current at the top of the sprite channel (the ionosphere) as a function of time.



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