Optimal Income Tax Schedules under Action Revelation

Jonathan Hamilton

and

Steven Slutsky

Department of Economics
Warrington College of Business
University of Florida
Gainesville FL 32611-7140 USA

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ABSTRACT

Most income tax systems rely on reporting after individuals earn income. Most theoretical tax models utilize the revelation principle and have individuals report types *ex ante*, before effort. With an infinite population, action revelation and announcement revelation do not differ significantly. However, with a finite number of individuals, a single individual's misrevelation is detectable, and the government can revise tax schedules after detecting misrevelation. Piketty showed that any full-information outcome is attainable under action revelation. However, the off-equilibrium allocations used to sustain full-information outcomes on the equilibrium path may not be optimal for the government to actually carry out—the government must commit to suboptimal allocations. We study outcomes for different levels of commitment under action revelation. With full commitment, distorted outcomes may arise when budget imbalance would have arisen off the equilibrium path. The distortion pattern can differ from the standard Mirrlees-Stiglitz pattern. Both types may be subsidized (or taxed) on the margin. The outcome under action revelation is weakly dominated by that under announcement revelation. Somewhat surprisingly, without full commitment, the government can sometimes do better using action revelation. Action revelation may increase the government's ability to commit to off-equilibrium policies and thus may allow it to redistribute more in equilibrium.

1. Introduction

Most income tax systems use *ex post* income reporting where individuals file tax returns after choosing labor effort. In contrast, most theoretical models of income taxation utilize the revelation principle and hence have individuals reporting types *ex ante*, before working. We call the former approach **action revelation** and the latter **announcement revelation**. With an infinite population—whether with a continuum of types (Mirrlees [1971]) or a finite set of types (Stiglitz [1982])—the government cannot detect misrevelation by a single individual. It announces tax schedules *ex ante* and does not revise them if anyone misreveals. Whether revelation is by action or announcement does not affect the optimal tax schedules.[1]

Instead, suppose that the number of individuals is finite and the government knows the true pattern of types but not any particular individual's type. If even one individual misreveals, the planner at some stage will detect that the revealed pattern of types does not match what it knows for certain to be the actual pattern.[2] If different types pay different taxes, then any misrevelation causes either a surplus or a deficit that the government would notice. Knowing that someone misrevealed (even though it does not know who), the government could condition the bundles it offers each individual on others' revelations as well as her own. Indeed, when misrevelation causes a deficit, to maintain budget balance, the government must have the taxes of at least some individuals depend on what others reveal.

In an economy with a finite population using action revelation, Piketty [1993] considered the use of generalized tax schedules where each individual's tax depends on others' incomes. He showed that the government not only could do better than the standard Mirrlees-Stiglitz second best, but could attain any full-information outcome. However, Hamilton and Slutsky [2007] showed that, to sustain many full-information outcomes in equilibrium, the government must announce tax schedules that do not balance the budget off the equilibrium path. Such announcements would not be feasible when a deficit occurs and hence would not be credible. For announcement revelation, they show that the result that any full-information outcome is attainable remains valid even if budget balance is required on and off the equilibrium path.

Not having a deficit in any circumstance may be necessary for a system of generalized taxes to be credible but it is not sufficient. Individuals will not believe feasible threats if the government cannot commit to them and it is not in the government's interest to carry them out. If the government announces off-equilibrium allocations that run a surplus to sustain a full-information outcome in equilibrium, it will not be optimal for the government to actually enact them. Attaining full-information outcomes would then require that the government be able to commit itself to making those suboptimal allocations. When the government cannot fully commit, such tax schedules would be revised, and the appropriate solution concept incorporates renegotiation-proofness. In that case, the government no longer has the role of a mechanism designer standing outside the game setting the rules under which individuals will play. It is now another player in the game, able to make certain moves but subject to the restrictions of optimizing at each of its information sets.[3]

We study the outcomes for different budget balance requirements off the equilibrium path under action revelation.[4] Results differ significantly from those under announcement revelation. Distorted outcomes may arise, in particular when budget imbalance would be needed off the equilibrium path. The pattern of distortions can differ from the Mirrlees-Stiglitz pattern in which the higher ability type faces no marginal distortion and the lower ability type is taxed on the margin. In some circumstances, it is optimal to have both types face negative marginal taxes (or positive marginal taxes). We also compare outcomes under announcement revelation and action revelation and show that neither approach dominates the other.

Section 2 specifies the model and section 3 considers some benchmarks for comparison.[5] Sections 4 and 5 present results for full commitment and renegotiation proofness. Finally, section 6 offers some conclusions.

2. The Model

(A) Structure

The economy has a finite number n of individuals of two different types. Let n^i (for i = 1 or 2) denote the number of each type. Each individual consumes a net trade bundle X^i with two components, the first being labor income and the second consumption. A linear production technology determines feasible pairs of net trades, $p \cdot (n^1X^1 + n^2X^2) \le 0$. Without loss of generality, we can specify the units of measurement so that $p = (-1,1)$. If $p \cdot X^i = 0$, then that net trade bundle involves no redistribution across types.

Each individual has a utility function $U^i(X^i)$ over the net trade vector. The utility functions can differ across types because of taste, productivity, or endowment differences. We make the following assumptions about preferences:

(i) $U^i(X^i)$ is quasi-concave and continuously differentiable. Since labor X_1^i is a bad, $\partial U^i/\partial X_1^i < 0$ and $\partial U^i/\partial X_2^i > 0$.

(ii) $\lim_{X_2^i \to 0} \dfrac{\partial U^i}{\partial X_2^i} \to \infty$. Thus, for any bundle X with $p \cdot X < 0$, there is an $\hat{X}_2 > 0$ such that $U^i(X) = U^i(0, \hat{X}_2)$.

(iii) Individuals have a maximum possible labor supply $\overline{L}$ and thus have maximum possible pre-tax incomes $\overline{X}_1^1$ and $\overline{X}_1^2$.[6] Furthermore, $\lim_{X_1^i \to \overline{X}_1^i} \frac{\partial U^i}{\partial X_1^i} \to -\infty$.

From these assumptions, we can characterize the indifference curves. Indifference curves slope up ($MRS^i(X) = -\frac{\partial U^i / \partial X_1^i}{\partial U^i / \partial X_2^i} > 0$). The indifference curve through any bundle X^* with $p \cdot X^* < 0$ must cross the line $p \cdot X = 0$ twice—at a point where $X_1 < X_1^*$ and at a point where $X_1^* < X_1 < \overline{X}_1^i$.

Next, we assume that neither consumption nor leisure is inferior. Consider lines $p \cdot X = K$ and let $X^i(K)$ be the bundle that solves $\text{Max } U^i(X)$, s.t. $p \cdot X = K$. Then,

(iv) If $K' > K''$, then $X_1^i(K') \leq X_1^i(K'')$ and $X_2^i(K') \geq X_2^i(K'')$.

In addition, we make a standard single-crossing assumption across types:

(v) At any bundle X, $MRS^1(X) > MRS^2(X)$ and hence $\overline{X}_1^1 < \overline{X}_1^2$.

Assumption (v) holds in the standard case in which individuals differ only in ability with type 2 more able than type 1.

(B) Timing of Actions

Nature moves first and determines each individual's type. The government learns the preferences and the exact number of each type, but it does not learn the type of any particular individual. All individuals know their own types and the total numbers of each type.[7]

The government will choose policies in the second and fourth stages to seek to implement a particular outcome. We assume that the equal treatment property is satisfied in any allocation that the government wishes to implement so that every individual earning the same income receives the same allocation. Then any outcome that the government wishes to implement is a pair of bundles $\left((X_1^1, X_2^1), (X_1^2, X_2^2)\right)$. In the second stage, the government announces a menu of provisional tax policies conditional on the vector of labor incomes that individuals report in the next stage. That is, for each possible vector of incomes, $X_1 \equiv \left(X_1^1, X_1^2, \ldots X_1^n\right)$, the government announces the provisional consumption assigned to each individual, $Y_2\left(X_1\right) \equiv \left(Y_2^1(X_1), Y_2^2(X_1), \ldots Y_2^n(X_1)\right)$. These provisional consumption levels vary with the entire vector of incomes and not just each individual's own income. The government may modify these bundles later in the game subject to some restrictions. In order for these provisional tax policies to implement the pair $\left(\left(X_1^1, X_2^1\right), \left(X_1^2, X_2^2\right)\right)$, any vector X_1 that has n^1 individuals choosing X_1^1 and n^2 individuals choosing X_1^2 must assign X_2^1 to the individuals choosing X_1^1 and X_2^2 to the individuals choosing X_1^2.[8] Any bundle to be implemented that is feasible and efficient must exactly balance the budget. That is:

$$n^1 X_2^1 + n^2 X_2^2 = n^1 X_1^1 + n^2 X_1^2 \qquad (1)$$

In addition, there are restrictions related to budget balance off the equilibrium path as well. These depend on what revisions the government can make to its provisional announcements and are presented below.

In the third stage, after observing the government's provisional tax policies, individuals simultaneously select their incomes which the government observes and are fixed for the remainder of the game. In making these choices, individuals are aware of what deviations, if any, the government will make in the fourth stage. The government wants the equilibrium of this game of individual choices to be unique. To ensure this, there must be restrictions on the $Y_2(X_1)$ provisional policies announced by the government in stage 3. These restrictions are in effect incentive compatibility constraints. With a finite number of individuals, the Mirrlees-Stiglitz self-selection constraints are not appropriate since the government detects deviations by even one person. Following Piketty [1993], we assume that desired behavior is induced by dominance solvability rather than the stronger notion that it is a dominant strategy. Our problem is more complicated since the government moves both before and after individual revelation. Individuals observe the provisional policies but also react to their beliefs of what adjustments the government will make after revelation. Since the final stage adjustments occur after individuals' income decisions, they cannot be directly constrained to yield dominance solvability.

To specify this formally, assume for now that the government can prevent any individual from selecting an income other than X_1^1 or X_1^2. We consider below how to do this under different possibilities of the modifications the government can make in stage 4. Hence, the crucial concern is whether individuals choose the income intended for their type or that for the other type. The government's problem then reduces to assigning consumptions to people who earn these incomes conditional on the number who choose each level. Let N^j be the total number who choose X_1^j and let $N^k = n - N^j$, for $j \neq k$, be the number who choose X_1^k, and let $X_2^j\left(N^j\right)$ and $X_2^k\left(N^j\right)$ be the consumptions assigned to the two income levels as functions of the number who choose each one. From the perspective of any particular individual, let $\hat{N}^j$ be the number of other individuals who choose income X_1^j. Let $X_2^i\left(N^j,Y\right)$ be the consumptions that someone choosing each type's income expects to receive after stage 4 as a function of how many chose each income level and the provisional policies from stage 2. Then dominance solvability holds if the following conditions are satisfied for j equal to either 1 or 2:

$$U^j(X_1^j,X_2^j(\hat{N}^j+1,\,Y)) \geq U^j(X_1^k,X_2^k(\hat{N}^j,Y)), \qquad 0 \leq \hat{N}^j \leq n-1 \qquad (2)$$

$$U^k(X_1^k,X_2^k(\hat{N}^j,Y)) \geq U^k(X_1^j,X_2^j(\hat{N}^j+1,Y)), \qquad n^j \leq \hat{N}^j \leq n-1 \qquad (3)$$

Condition (2) imposes that choosing X_1^j is a dominant strategy for type j: no matter how many others have chosen X_1^j, it is better for type j to choose X_1^j than X_1^k. From condition (3), it is then a dominant strategy for type k to choose X_1^k given that every type j has chosen X_1^j.

In the fourth stage, having learned the vector X_1 that individuals chose, the government specifies the actual taxes it will levy. We specify these as bundles $X_2(X_1)$. We consider two possibilities for what deviations the government can make from its second stage provisional announcements. If no deviations are allowed, the government can fully commit to feasible announcements. Formally, this imposes the restriction:

$$X_2(X_1) = Y_2(X_1), \qquad (4a)$$

A second possibility is renegotiation proofness where changes can be made only if no one objects.[9] Since the incomes are now set, the only changes that can be made are in the consumptions. The no-objection restriction is then:

$$X_2^i(X_1) \geq Y_2^i(X_1), \qquad \text{for all i} \qquad (4b)$$

Anyone can block a change from the provisional announcement if it lowers his or her consumption.

(C) Social Preferences

This game has n + 1 strategic players: n individuals and the government. Individuals' utility functions define their payoffs. The government's preferences derive from a weighted utilitarian welfare function. Given the vector of individual incomes, the government forms beliefs about the types (or mix of types, if pooling occurs) of individuals who choose a particular income. Let $i\left(X_1^i \middle| X_1\right)$ be the type that the government believes an individual who has chosen income X_1^i to be when the complete vector is X_1 (for i = 1 or 2). Let α_1 and $\alpha_2 = 1 - \alpha_1$ be the social welfare weights given to individuals believed to be types 1 and 2 respectively.[10] Then we assume that social welfare is $W\left(X_1, Y_2\left(X_1\right)\right) = \sum_{i=1}^{n} \alpha_i N_i U^i\left(X_1^i, Y_2^i\left(X_1\right)\right)$.[11] We assume that individuals know this function and can accurately predict what policies the government will implement in stage 4 conditional on the chosen vector X_1.

(D) Solution Concept

Since, in stage 3, individuals observe the provisional government policies before revealing their type and**,** in stage 4, the government observes individuals' revelations before implementing a final policy, the appropriate solution concept is sequential equilibrium (Kreps and Wilson [1982]). The player who moves at any information set maximizes given her beliefs, and those beliefs must be consistent with equilibrium strategies, where possible.

In stage 4, each vector X_1 defines a different information set for the government with $i\left(X_1^i \middle| X_1\right)$, i = 1, ..., n, its beliefs about individuals' types. In forming these beliefs, the government presumes that this vector arises from minimum deviations from the incomes which it intended individuals to choose. If the vector matches exactly what was intended, then the government believes each individual acted as desired and does not assume that offsetting deviations occurred. If all individuals chose intended incomes but the numbers do not match the distribution, then the government believes that everyone acted as intended except for the minimum number who must have deviated.[12]

3. Benchmarks

It is useful to compare our equilibrium outcomes to some standard benchmarks. First, if the government has full information about individuals' types, it implements the solution to:

$$\textbf{(FI)} \qquad \underset{X^1, X^2}{\text{Max}} \quad \alpha n^1 U^1(X^1) + (1-\alpha) n^2 U^2(X^2) \qquad \text{s.t.} \qquad p \cdot (n^1 X^1 + n^2 X^2) \le 0.$$

Choice of the parameter α allows this problem to describe any undistorted allocation. Let $X^i(n^1, \alpha)$ denote the solution to **FI** for any n^1 and α. There exists an α^o such that the solution to **FI** entails no redistribution $(p \cdot X^1(n^1, \alpha^o) = p \cdot X^2(n^1, \alpha^o) = 0)$. For $\alpha > \alpha^0$, transfers from type 2's to type 1's occur $(n^1 p \cdot X^1(n^1, \alpha) = - n^2 p \cdot X^2(n^1, \alpha) > 0)$ with the reverse for $\alpha < \alpha^0$.

As shown by Piketty [1993], under action revelation, any solution to **FI** can be sustained with generalized tax schedules when the government does not know types of specific individuals. Piketty placed no feasibility or budget balance restrictions on off-equilibrium bundles. As Hamilton and Slutsky [2007] show, sustaining some full information allocations might only be possible if the budget does not balance in some off-equilibrium situations. To see this, consider an economy with just two individuals, one of each type. Let $\left(X^1, X^2\right) \equiv \left(\left(X_1^1, X_2^1\right), \left(X_1^2, X_2^2\right)\right)$ denote the government's desired allocation which is assumed to be a constrained efficient allocation. Assume for now that neither individual chooses an income level other than X_1^1 or X_1^2. Dominance solvability over these two incomes can be achieved in two ways. One is for individual 1 to prefer to earn X_1^1 no matter which income individual 2 chooses and then for individual 2 to prefer to earn X_1^2 given that individual 1 earns X_1^1. The other is for 2 to prefer earning X_1^2 regardless of 1's decision and then for 1 to choose X_1^1 when 2 earns X_1^2. Note that, for either way, three conditions are needed. Two come from the requirement that the desired allocation should be a Nash equilibrium so that each must choose the appropriate income when the other does. The third comes from the requirement that the desired allocation must be the unique Nash equilibrium. Hence, at least one individual must choose the appropriate income even when the other does not. When off-equilibrium feasibility or budget balance is imposed, there are circumstances in which either of these yields a superior outcome. Without off-equilibrium restrictions, only one of the ways is needed; Piketty only needed dominance solvability with the lower ability type having a dominant strategy for his results.

The following Lemmas specify different possibilities for dominance solvability depending upon the direction of the income transfer and the type with the dominant strategy. Denote the bundle the government assigns when both choose X_1^2 by $X(0)$, and when both choose X_1^1 by $X(2)$.

Lemma 1: If type j receives a transfer from type k, then

$$U^j\left(X_1^j,X_2^j\right) > \max\left\{U^j\left(X_1^k,X_1^k\right),U^j\left(X_1^k,\hat{X}_2^k\right)\right\} \text{ where } U^k\left(X_1^k,\hat{X}_2^k\right) \equiv U^k\left(X_1^j,X_2^j\right).$$

This lemma has implications for when the budget can always be balanced if the taxed individual has the dominant strategy. Since $U^j\left(X_1^j,X_2^j\right) > U^j\left(X_1^k,X_1^k\right)$, having a balanced budget if both choose the taxed individual's intended income always ensures that the recipient (regardless of which direction redistribution takes) would strictly prefer to earn his intended income than that of the taxed individual. If $\hat{X}_2^k \le X_1^k$, then having a balanced budget when both choose the taxed individual's intended income would also ensure that the taxed individual prefers his own intended income to that of the recipient individual when the recipient chooses the wrong income. If, however, $\hat{X}_2^k > X_1^k$, then a deficit would be needed when both choose the taxed individual's intended income to ensure that the taxed individual would not prefer the recipient's intended income. Since $U^j\left(X_1^j,X_2^j\right) > U^j\left(X_1^k,\hat{X}_2^k\right)$, the no-deficit constraint off the equilibirum path would bind with respect to the taxed individual.

Lemma 2: Assume that the transfer is from type 2 to type 1.

(a) This allocation is sustainable by dominance solvability with 1 having a dominant strategy to choose X_1^1 with $X(0)$ satisfying budget balance and

(i) with $X(2)$ satisfying budget balance if $U^1\left(X_1^1, X_1^1\right) > U^1\left(X_1^2, X_2^2\right)$ and

$U^2\left(X_1^2, X_2^2\right) > U^2\left(X_1^1, X_1^1\right)$;

(ii) with a deficit required at $X(2)$ if $U^1\left(X_1^2, X_2^2\right) > U^1\left(X_1^1, X_1^1\right)$; and

(iii) with a surplus required at $X(2)$ if $U^2\left(X_1^1, X_1^1\right) > U^2\left(X_1^2, X_2^2\right)$.

(b) This allocation is sustainable by dominance solvability with 2 having a dominant strategy to choose X_1^2 and with budget balance at X(0) and X(2) iff $U^2\left(X_1^2, X_1^2\right) \geq U^2\left(X_1^1, X_2^1\right)$. If $U^2\left(X_1^2, X_1^2\right) < U^2\left(X_1^1, X_2^1\right)$, then an off-equilibrium deficit is required at $X(0)$ to have dominance solvability of this type.

Thus, from (a), when taxes are designed so that the lower ability person has the dominant strategy, budget balance can be achieved if, at the income level intended for that type, the no-tax 45° line goes through the gap between the two types' indifference curves through the bundle intended for the higher ability type. A deficit is required if that indifference curve for type 1 is above the 45° line at that income, and a surplus is required if type 2's indifference curve through that bundle is below the 45° line at that income level. On the other hand, if taxes give the higher ability individual the dominant strategy, then there is off-equilibrium budget balance if and only if type 2's indifference curve through the bundle intended for type 1 is below the no-tax 45° line at the income level intended for the type 2. If this does not hold, then a deficit is required. For this direction of dominance solvability, a surplus is never necessary.

When transfers are from 2 to 1, to show that there exist circumstances in which budget balance must be violated off the equilibrium path to sustain an **FI** allocation, we need to show that budget balance is inconsistent with either way of doing dominance solvability. From Lemma 2, this can happen in two ways. One is that deficits are required at both X(0) and X(2). The one at X(0) is needed for 2 to have a dominant strategy, and the one at X(2) is needed for 1 to have a dominant strategy. The other is that a surplus is needed at X(2) to insure that 2 chooses X_1^2 when 1 chooses X_1^2. The first way occurs if the conditions in (a(ii)) ($U^1\left(X_1^2, X_2^2\right) > U^1\left(X_1^1, X_1^1\right)$) and (b) ($U^2\left(X_1^2, X_2^2\right) < U^2\left(X_1^1, X_2^1\right)$) both hold. See Figure 1 for an example of when this happens. The second way occurs if the conditions in (a(iii)) ($U^2\left(X_1^1, X_1^1\right) > U^2\left(X_1^2, X_2^2\right)$) and (b) ($U^2\left(X_1^2, X_2^2\right) < U^2\left(X_1^1, X_2^1\right)$) both hold.[13]

When α is such that the transfer is from the less able to the more able, the possibilities for dominance solvability with budget balance are simpler.

Lemma 3: Assume that the transfer is from type 1 to type 2. If X(0) and X(2) satisfy budget balance, then 2 has a dominant strategy to choose X_1^2. Dominance solvability then holds iff $U^1\left(X_1^1, X_2^1\right) \geq U^1\left(X_1^2, X_1^2\right)$. If this does not hold, then a deficit at X(0) is required.

When transfers go from the lower ability type to the higher ability one, in a dominance solvable allocation with off-equilibrium budget balance, both types have a dominant strategy to choose their intended incomes.[14] However, if dominance solvability cannot be achieved with budget balance, then it is the requirement that type 1 chooses X_1^1 when type 2 chooses X_1^2 that is the binding constraint.

That off-equilibrium budget imbalances may be needed to sustain some full information outcomes is significant since deficits are not feasible even off the equilibrium path, so such stage 3 announcements cannot be implemented in stage 4 and will not be believed. Thus, feasibility conditions must be imposed on the provisional announcements and on the final taxes:

$$\sum_{i=1}^{n} Y_2^i(X_1) \le \sum_{i=1}^{n} X_1^i \qquad \text{for all } X_1 \tag{5}$$

$$\sum_{i=1}^{n} X_2^i(X_1) \le \sum_{i=1}^{n} X_1^i \tag{6}$$

Under full commitment, (6) follows from (5) and under renegotiation proofness, (5) is needed even though there can be later revisions because of the restriction of (4b).

A second benchmark is the Mirrlees-Stiglitz model where the government knows the distribution of types but not any individual's type, and it commits to taxes which depend only on each individual's own actions. To induce truthful reporting, the government offers everyone a choice from the same pair of net trade bundles, where the bundles satisfy self-selection constraints. The planner's optimization problem is:

(MS)

$$\begin{aligned} \underset{X^1, X^2}{\text{Max}} \ & \alpha n^1 U^1(X^1) + (1-\alpha) n^2 U^2(X^2) \\ \text{s.t.} \quad & U^1(X^1) \ge U^1(X^2) \\ & U^2(X^2) \ge U^2(X^1) \\ & p \cdot (n^1 X^1 + n^2 X^2) \le 0. \end{aligned}$$

Our next result establishes that generalized tax schedules with budget balance everywhere can sustain any allocation that satisfies the Mirrlees-Stiglitz self selection constraints.

Lemma 4: Consider an economy with one individual of each type. If the outcome satisfies the **MS** constraints, then this allocation can be sustained under generalized tax schedules with budget balance off the equilibrium path with the taxed type having a dominant strategy.

It then follows that, in a finite economy even with off-equilibrium budget balance, the government can do at least as well and often better than in the Mirrlees-Stiglitz model. Thus, while Piketty's strong result that all full information outcomes can be achieved with generalized taxes is not true when off-equilibrium budget balance is imposed, an important kernel remains. Using generalized taxes even with off equilibrium budget balance allows the government to do better than in the standard model whenever distortions must arise as shown in Theorem 1.

Theorem 1: If an allocation solves **MS** for some α, then there exists an allocation that weakly Pareto dominates it and is a dominance solvable equilibrium using generalized tax schedules that satisfy budget balance everywhere. If the **MS** allocation lies off the **FI** frontier, then the Pareto dominance is strict.

There are several important points about this result. First, as shown in the proof, the taxed individual has a dominant strategy, not the recipient as in Piketty. If transfers are very large so that the **MS** self-selection constraints do not hold at the **FI** allocation, then the recipient may have the dominant strategy. Second, since the dominance solvability conditions all hold with strict inequality, they continue to hold for slightly larger redistributions that violate the **MS** self-selection constraints. Hence, **FI** outcomes can be sustained with budget balance and generalized tax schedules when distortions are required in the Mirrlees-Stiglitz framework. Third, with more than one individual of each type, there is more flexibility in choosing consumptions off the equilibrium path, so the result continues to hold. Finally, note that the **MS** self-selection constraints are satisfied for α near α^{o} so that full information allocations with little redistribution can be sustained with generalized taxes and budget balance.

The next results specify sufficient conditions on preferences and the degree of redistribution so that sustaining the FI outcome when redistribution is from 2 to 1 requires either a deficit or a surplus at X(0) or X(2).

Theorem 2:

(A) For sufficiently large α, a deficit is required at X(0) to ensure dominance solvability with 2 having the dominant strategy.

(B) If there exists an $\hat{X}_1^1 > 0$ with $X_1^1(1,\alpha) \geq \hat{X}_1^1$, all α, then, for sufficiently large α, an off-equilibrium surplus at X(2) is required to sustain $X^i(1,\alpha),\ i = 1,2$ under dominance solvability. When such a surplus is required, then a deficit is needed at X(0) for dominance solvability with 2 having the dominant strategy. However, if budget balance were required, a no-surplus constraint at X(2) would be the binding constraint to ensure either type of dominance solvability.

(C) If $X_1^1(1,\hat{\alpha}) = 0$, for some $\hat{\alpha}$, then a deficit at X(2) is required to sustain the allocations $X^i(1,\alpha),\ i = 1,2$ for $\hat{\alpha} \leq \alpha \leq 1$ with 1 having the dominant strategy to get dominance solvability.

Several points deserve mention. First, which type has the dominant strategy can vary with the amount of redistribution. When little redistribution is done (α is near α^{o}), type 2 has the dominant strategy from Lemma 4. If, after significant redistribution, either the condition in part (B) holds or the condition in part (C) holds and the restrictions at X(2) are binding, then type 1 would have the dominant strategy. Second, the condition in (B) that a surplus is required when type 2 chooses the income intended for type 1 is satisfied if type 1's demand for leisure is unaffected by the size of the transfer $(\partial X_1^1(1,\alpha)/\partial\alpha = 0)$. Third, these conditions are sufficient but not necessary. For example, the sufficient condition in (C) for infeasibility is only one of many possibilities. If type 1's demand for consumption is unaffected by the transfer $(\partial X_2^1(1,\alpha)/\partial\alpha = 0)$, then the off-equilibrium path allocations must be infeasible for large α. Finally, when $X_1^1(1,\alpha) > 0$ for all α but is not bounded away from zero, it may be possible to sustain any full-information allocation with budget balance. If $U^1(X_1^1(1,\alpha),X_1^1(1,\alpha)) \geq U^1(X^2(1,\alpha))$ and $U^2(X^2(1,\alpha)) \geq U^2(X_1^1(1,\alpha),X_1^1(1,\alpha))$ for all α, the achievable utility possibility frontier with budget balance is identical to the full-information utility possibility frontier.

4. Equilibria under Full Commitment

Under full commitment, the government can commit to run a surplus and discard the excess revenue. Therefore, given assumption (ii), it is straightforward for the government to make it strictly dominated for any individual to reveal an income other than X_1^1 or X_1^2 by assigning a consumption of 0 to any income other than those two but assigning positive income to anyone who reveals X_1^1 or X_1^2. Dominance solvability then reduces to just conditions (2) and (3) being satisfied. In addition, the government cannot commit to infeasible policies so that (5) must be satisfied. That condition simplifies to:

$$N^1X_2^1(N^1) + N^2X_2^2(N^1) \leq N^1X_1^1(N^1) + N^2X_1^2(N^1), \text{ all } N^1 \tag{7}$$

The equilibrium of the game then reduces to being the solution to a simple optimization problem where X_1^1, $X_2^1(N^1)$, X_1^2, and $X_2^2(N^1)$ for all N^1 are chosen to maximize $\alpha n^1U^1(X^1(n^1)) + (1-\alpha)\, n^2U^2(X^2(n^1))$ subject to (2), (3), and (7). Note that, while the government chooses allocations for every N^1, only the bundles where $N^1 = n^1$ (the true number) enter the objective function. We call this the full commitment problem (**FC**).[15]

Under full commitment, the government can achieve any full-information outcome that has a balanced budget or a surplus off-equilibrium. If an off-equilibrium deficit is required, then the full-information outcome cannot be attained, and distortions must be imposed. Theorem 3 characterizes the equilibrium bundles in that case.

Theorem 3: Consider an economy with one individual of each type and assume that the no-deficit constraint binds off the equilibrium path.

(1) If redistribution is toward type 1, then, on the equilibrium path, both types consume bundles with distortions in the same direction where either

(a) type 1 has a dominant strategy; $MRS^1\left(X_1^1,X_2^1\right)>1$ and $MRS^2\left(X_1^2,X_2^2\right)>1$; and the transfer exceeds the full-information transfer at these income levels; or

(b) type 2 has a dominant strategy; $MRS^1\left(X_1^1,X_2^1\right)<1$ and $MRS^2\left(X_1^2,X_2^2\right)<1$; and the transfer is smaller than full-information transfer at these income levels.

(2) If redistribution is toward type 2, then type 2 has a dominant strategy: $MRS^1\left(X_1^1,X_2^1\right)=1$ and $MRS^2\left(X_1^2,X_2^2\right)>1$; and the transfer exceeds the full information transfer at these income levels.

While the distortion pattern in (2) is the same as in the Mirrlees-Stiglitz model, that is not true in (1). First, no matter which type has the dominant strategy, the type paying a positive total tax faces a distortion. Second, when the optimal tax system gives type 1 the dominant strategy, then both types are subsidized on the margin instead of being taxed. An increase in earnings for both types shifts 1's indifference curve through X_1^2 down and restores earning X_1^1 to be a dominant strategy for type 1. Which distortion pattern arises depends on what type of constraint binds. If a Nash equilibrium constraint binds, the Mirrlees-Stiglitz pattern arises. If a uniqueness constraint binds, the double distortion arises.

The utility possibility frontiers under full commitment and in the **MS** model diverge from the full information frontier in different ways. Brito et al. [1990] show that, in the **MS** model with finite types, the achievable utility possibility frontier coincides with the full information frontier over a connected range around the no-redistribution utilities. For extensive redistribution in either direction, distortions must be imposed and the achievable frontier lies below the full information frontier. There are two connected intervals of divergence, one for each direction of redistribution. In our model, the interval around the no-redistribution utilities where the **FI** and **FC** frontiers coincide is larger than in the **MS** model, possibly even covering the entire full information frontier. When redistribution is toward the more able type, if the frontier diverges from the full information one, as in the **MS** model, it will do so in a connected interval. However when redistribution is toward the less able type, there can be a segment where the frontier diverges from the full information one but then with more redistribution, the two frontiers again coincide. With sufficient redistribution, exact off- equilibrium budget balance may not sustain a full information allocation, but sometimes a surplus is required and sometimes a deficit. When a deficit is required, the frontiers diverge, but a surplus is sustainable under full commitment, and the outcome is again efficient.

For redistribution from 2's to 1's, when there is one individual of each type, an undistorted allocation cannot be sustained under full commitment when deficits would be needed at X(0) and X(2). Theorem 4 shows that increasing the number of individuals to two of each type may relax or tighten the government's no-deficit constraints and thereby may increase or decrease the set of undistorted allocations sustainable under full commitment.

Theorem 4: If $n^1 = n^2 = 2$, the set of undistorted allocations which can be sustained can be larger or smaller than when $n^1 = n^2 = 1$.

Hence, the failure to sustain all undistorted allocations is not an artifact of having just two individuals. With more individuals, it is even possible that fewer undistorted allocations can be sustained. With more individuals, there are more off-equilibrium budget balance restrictions. However, incentive constraints are relaxed. Having more individuals adds bundles between $X^1(n^1, \alpha)$ and $X^2(n^1, \alpha)$; these choices give the government additional flexibility. This can help overcome the preference of type 2s for type 1's full-information bundle with significant redistribution toward type 1s. Which effect dominates depends upon preferences as shown in the proof for the case with four individuals. Whether type 1's indifference curve through the no-redistribution bundle X(4) intersects the vertical line through $X_1^2(1, \hat{\alpha})$ above or below the $p \cdot X = 0$ line depends upon how much curvature the indifference curves have and how close $X_1^1(1, \hat{\alpha})$ and $X_1^2(1, \hat{\alpha})$ are to each other. The more they differ, the more likely it is that more efficient redistribution can be done when there are more individuals.

5. Renegotiation-Proof Equilibria

To analyze this case, we must find conditions for renegotiation-proofness and then, given that renegotiation-proofness is satisfied, specify conditions for dominance solvability so that the renegotiation-proof equilibrium is unique. Conditions for renegotiation-proofness are more complicated than under full commitment since the government can change consumptions after individuals have chosen their incomes. In stage 4, for each X_1, the government will adjust taxes so that the budget exactly balances. A deficit is infeasible and a surplus could be distributed among individuals raising the utilities of both types and hence raising social welfare. Knowing the government's beliefs, individuals can predict what the government will do in the final stage.

They will thus not believe that policies such as giving zero consumption to individuals who choose a level of income other than X_1^1 or X_1^2 will be carried out. Furthermore, any outcome that arises after readjustments could have been announced initially in stage 2. It thus would have been feasible under with full commitment and exact budget balance, in contrast to only ruling out deficits. Conversely, any solution under full commitment and exact budget balance is also possible with renegotiation proof re-optimization. If stage 2 announcements exactly balance the budget, only the consumption components can change in stage 4 since incomes are fixed. Budget balance implies that raising one type's consumption at any given X_1 requires lowering the other type's consumption which violates condition (4b) of renegotiation proofness. It immediately follows from these arguments that exact budget balance in all situations is equivalent to renegotiation proofness.[16]

Lemma 5: A menu of provisional tax policies announced in stage 2 is renegotiation proof if and only if condition (5) holds with equality.

Thus, renegotiation proofness differs from full commitment only because exact budget balance must hold. The government cannot run either surpluses or deficits off the equilibrium path. In part, this result depends upon the assumption that there are only two components to the bundles. Once income is fixed, only consumption can vary, and the two types have identical preferences over changes in consumption alone. In this case, it is relatively easy to make any reoptimization infeasible.[17]

Next, consider conditions for dominance solvability. It is no longer trivial to ensure that no individual will choose an income other than one of the equilibrium values. Unlike the full commitment case, budget balance must hold in all situations. The government cannot simply threaten to punish anyone who chooses an income level that no one should select by giving them low consumption without violating budget balance—increasing their consumption *ex post* would be a Pareto improvement. It is necessary to specify what is done with the consumption taken from such individuals. In Appendix B (available on-line) for the case of one individual of each type, we present generalized tax schedules defined for all income distributions that balance the budget in all circumstances and that yield strong incentives to avoid non-equilibrium incomes. Given these tax schedules, we then present a sufficient condition on the allocations that the government seeks to implement so that dominance solvability rules out anyone choosing an income level other than those in allocations on the equilibrium path.

After ruling out non-equilibrium incomes, renegotiation proofness requires selecting among equilibrium incomes in a dominance solvable way just as under full commitment. Therefore, renegotiation proof outcomes can be found as the solution to the same optimization problem determining full commitment outcomes, except condition (5) must hold with equality. It is now straightforward to find the renegotiation proof equilibria and to compare them to those under full commitment. For redistributions allowed by the condition in Appendix B, they are identical when the full information outcome can be sustained by off-equilibrium balanced budgets or deficits. The full commitment equilibrium would impose off-equilibrium budget balance in these cases, so it would also be renegotiation proof. When an off-equilibrium surplus is needed to sustain a full information optimum, then outcomes under renegotiation proofness are inferior to those under full commitment, since the government can commit to a surplus but must balance the budget if it cannot commit. For one individual of each type, as shown in Theorem 2B and the discussion following, there exist preferences for which, at α near 1, a surplus off the equilibrium path is required to sustain the full-information outcome. These allocations are attainable under full commitment but not under renegotiation proofness. However, for some preferences, a surplus is never needed to sustain full-information allocations—then outcomes under full commitment and renegotiation proofness are the same for all α.

When the no-surplus constraint binds under renegotiation proofness, Theorem 5 describes the resulting pattern of distortions. Unlike the situation when the no-deficit constraint binds, the pattern of distortions is the same as in the standard Mirrlees-Stiglitz model.

Theorem 5: Consider an economy with one individual of each type with redistribution from type 2s to type 1s. In a unique renegotiation proof equilibrium, if the no-surplus constraint binds, then, on the equilibrium path, the type 2 bundle is undistorted, but for type 1's, $MRS^1 < 1$. In addition, the government is doing insufficient redistribution in the sense that, if the incentive constraint were removed but incomes were fixed, the government would gain by increasing the consumption transfer from 2's to 1's.

Finally, it is worthwhile to compare outcomes under announcement and action revelation. With full commitment, the optimization problem for action revelation is **FC**. Announcement revelation is essentially the same problem without the constraint that an individual's income be equal across information sets. Hamilton and Slutsky [2007] show that, under announcement revelation and full commitment, the government can sustain any solution to **FI** while Theorems 3 and 4 show that this is not true under action revelation. Hence, with full commitment, the government does at least as well (and sometimes strictly better) under announcement revelation than under action revelation.

With renegotiation proofness, it is no longer true that outcomes under announcement revelation are as least as good as those under action revelation. Under action revelation, the only implication of renegotiation proofness is that budget balance must hold in all situations. Under announcement revelation, renegotiation proofness has more implications, and the equilibrium cannot be found as the solution to an optimization problem. With one individual of each type, individuals compare solutions to three separate stage 4 optimizations for their stage 3 decisions. The added flexibility under announcement revelation may benefit the government in each of these optimizations individually but harm it in the overall problem. Achieving a better result in one off-equilibrium optimization tightens the prior stage revelation constraint, and this sometimes hurts the government. In effect, the additional constraints of action revelation act as a form of additional commitment, limiting re-optimization. For the two-argument utility functions considered here, this can be powerful. Once income is fixed, especially off the equilibrium path, the planner has little flexibility in re-optimizing since only consumption, constrained by budget balance, can be varied.

For both types of revelation, there exist α near α^0 for which full-information allocations are sustained. The set of such α can be larger or smaller under action revelation. Under announcement revelation, the set of sustainable undistorted allocations is always a proper subset of the full-information Pareto frontier. As discussed after Theorem 1, there exist preferences for which the entire full-information frontier is sustainable under action revelation, and thus, for some α, action revelation yields greater social welfare.

In other circumstances, announcement revelation does better. Assume one individual of each type, and let β denote the highest value of α such that the solution under announcement revelation is on the full-information frontier. When individuals pool, re-optimization under renegotiation proofness will assign both types bundles on the $p \cdot X = 0$ line between the optimal points for the two types on that line. The most redistribution occurs when type 1's best bundle is assigned. Consider type 1's indifference curve through $X^2(1, \beta)$, the undistorted bundle for the type 2 given β. Preferences can be such that the type 1 indifference curve lies above the

$p \cdot X = 0$ line at $X_1^1(1,\beta)$. Action revelation then sustains a smaller set of full-information allocations since the undistorted bundle for $\alpha = \beta$ cannot be sustained under it. See Figure 2.

5. Conclusions

If there is a finite set of individuals and the government knows precisely the distribution of types, then optimal taxes depend significantly on several factors that are not relevant in the standard Mirrlees-Stiglitz framework. These include whether individuals reveal their type by actions or announcements, whether the government can run a surplus off the equilibrium path or must exactly balance the budget in all circumstances, and whether the government can commit to policies specified before individual revelation or can adjust those policies after revelation as long as no individual objects. These factors interact. Under announcement revelation with full commitment, which off-equilibrium budget restriction is imposed is irrelevant. Even with the strong constraint that the budget must exactly balance in all circumstances, the government can sustain any full information Pareto optimal allocation. If instead the government can make adjustments subject to restrictions of renegotiation proofness, then not all full information Pareto optima can be achieved but no individual ever faces marginal distortions. Under action revelation, there is complete alignment between the different levels of commitment and the restrictions on budget balance; imposing a no-deficit constraint is equivalent to full commitment and imposing exact budget balance is equivalent to renegotiation proofness. If the government needs to run a surplus off the equilibrium path to sustain some full information optimum, then in the renegotiation proof equilibrium, the same pattern of distortions arise as in the standard model: the net taxed individual faces no marginal distortion and the net subsidized individual is taxed on the margin. When the government needs an off-equilibrium deficit to sustain a full information optimum, then in both full commitment and renegotiation proof equilibria, the pattern of distortions is quite different from in the standard model. Both types are distorted with a subsidy (tax) on the margin to induce them to work more (or less) than with lump sum taxation.

Under action revelation, off-equilibrium budget restrictions reduce the set of implementable undistorted allocations below what Piketty found without those restrictions where every full information optimum was implementable. However, the set of implementable undistorted allocations is still larger than in the Mirrlees-Stiglitz model.

Surprisingly, there is ambiguity as to whether more full information allocations can be implemented with announcement or action revelation. Although action revelation imposes extra constraints on the government, only when there is full commitment with just a no-deficit constraint off the equilibrium path do the equilibria under announcement revelation weakly dominate those under action revelation. When re-optimization is allowed and exact budget balance is imposed, then, depending on both individual and social preferences, either method might allow more undistorted redistribution than the other.

That action revelation can be superior to announcement revelation in some sense violates the revelation principle. The nature of the violation differs from that of Bester and Strausz [2000] in a principal-agent model with multiple agents and imperfect commitment by the principal. They consider the revelation principle to be violated if the size of the message space is greater than the number of types, which occurs with partial pooling. In our case, the size of the message space under both action and announcement revelation equals the number of types.[18] The difference between the two revelation methods is whether consistency is required across on- and off-equilibrium allocations, a factor not considered by Bester and Strausz.

Finally, although we have focused on the optimal income taxation problem, there are important lessons from this analysis for any mechanism design problem with a finite number of agents. If only to avoid imposing infeasible actions off the equilibrium path, policies affecting one agent cannot be independent of what other agents do. With such dependence, it becomes significant how agents reveal their attributes, by announcement or by action. The principal may also be able to alter policies after revelation. If so, the restrictions of renegotiation proofness we considered here may be an appropriate and realistic possibility lying between complete commitment and no commitment.

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[1] One possible difference is that action revelation prevents randomizing both consumption and earned income.

[2] Of course, when there is "noise" in the structure of the problem (for example, if the government does not know the precise distribution of types), then any outcome may be on the equilibrium path with positive probability and detecting misrevelation becomes much more complicated.

[3] Gaube [2012] and Simon [2012] consider optimal taxation in intertemporal settings with different levels of government commitment. Without full commitment, the government becomes a player instead of a mechanism designer.

[4] Hamilton and Slutsky [2012] study how different levels of commitment and budget balance requirements correspond to differences in the legal system regarding separation of powers and judicial review.

[5] Proofs of the results in the text are in Appendix A (available on-line).

[6] This assumption means that any redistribution problem will be bounded since the maximum amount of redistribution is limited. The government could transfer no more than $\bar{X}_1^1$ from a type 1and and no more than $\bar{X}_1^2$ from a type 2.

[7] Except for an individual's knowledge of her own type, individuals and the government have the same information about the aggregate distribution. This is not a model such as Abreu and Sen [1991] where individuals have complete information about the type of every individual.

[8] For action revelation to work, $X_1^1 \neq X_1^2$ must hold for the government to distinguish between types. As long as leisure is strictly increasing in lump-sum income when $X_1 > 0$, this occurs for at most one full-information allocation. Since the planner could sustain full-information allocations that are arbitrarily close to that one, we ignore this case in our formal analysis. Note that this issue only arises for transfers toward type 2, given normality of leisure.

[9] See Farrell and Maskin [1989] and Fudenberg and Tirole [1990] for discussions and applications of renegotiation proofness.

[10] Under action revelation as opposed to announcement revelation, since no individual can be said to have directly lied, we do not allow these weights to vary with whether the government believes someone has misrevealed as in Hamilton and Slutsky [2012].

[11] The government might gain by acting as if its preferences differ from its true ones in order to affect what income individuals will choose in stage 3. We assume that the government cannot commit to a false welfare function and thus must act in stage 4 according to its true preferences.

[12] Assuming that any information set off the equilibrium path arises from the minimal number of deviations satisfies the limit restriction of Kreps and Wilson [1982, p. 875]. In Appendix B, we consider beliefs after income choices other than X_1^1 or X_1^2.

[13] The condition in (b) that $U^2\left(X_1^2, X_1^2\right) < U^2\left(X_1^1, X_2^1\right)$ must hold from normality.

[14] The condition $U^1\left(X_1^1, X_2^1\right) \geq U^1\left(X_1^2, X_1^2\right)$ must hold for the desired allocations to be a Nash equilibrium. When this does hold, then $U^1(X^1) > U^1(X_1^2, X_1^2)$, so that type 1 also has a dominant strategy (see footnote 1 in Appendix A).

[15] We write conditions (2) and (3) with weak inequalities. Typically, dominance solvability conditions involve iterated elimination of strictly dominated strategies. When a game is dominance solvable in that sense, it has a unique Nash equilibrium. If one eliminates weakly dominated strategies instead, the process may eliminate some Nash equilibria when multiple equilibria exist (see Gretlein [1983] for an explanation of the difficulties that arise from eliminating weakly dominated strategies). Here, we want the mechanism to have a unique equilibrium and intend to eliminate only strictly dominated strategies. However, since these conditions serve as constraints in an optimization problem, if we specify strict inequalities, the feasible set would not be closed, potentially leading to difficulties. As developed in the proofs, most of these conditions hold with strict inequality (see, for example the analysis just prior to Theorem 1 with one individual of each type). At least one constraint holds with equality. This is similar in effect to having standard self-selection constraints hold with weak inequalities. When individuals are indifferent between acting as the government intends or deviating, we assume they act as intended. A bundle within an arbitrarily small ε of the solution exists which would make the condition hold with strict inequality.

[16] While the limited degree of commitment gives the government the opportunity to revise consumption levels after individuals choose their incomes, revisions may never be observed. A government that forsees its future decisions could simply announce them at stage 2 and then have no desire to make revisions. The limited commitment acts as a constraint on stage 2 announcements so that the government does not make revisions.

[17] However, if the government cannot differentially tax different consumption goods and can only tax income, then a composite commodity theorem would reduce the many consumption goods to a single composite good, making this two-good model appropriate.

[18] This is clearly true for announcement revelation. For action revelation, the message space might seem to be any feasible income level for each type which is a continuum. However, as considered in Appendix B, the government can specify tax schedules so that only one of two levels of income will ever be selected by any individual and thus the size of the message space reduces to the number of types.

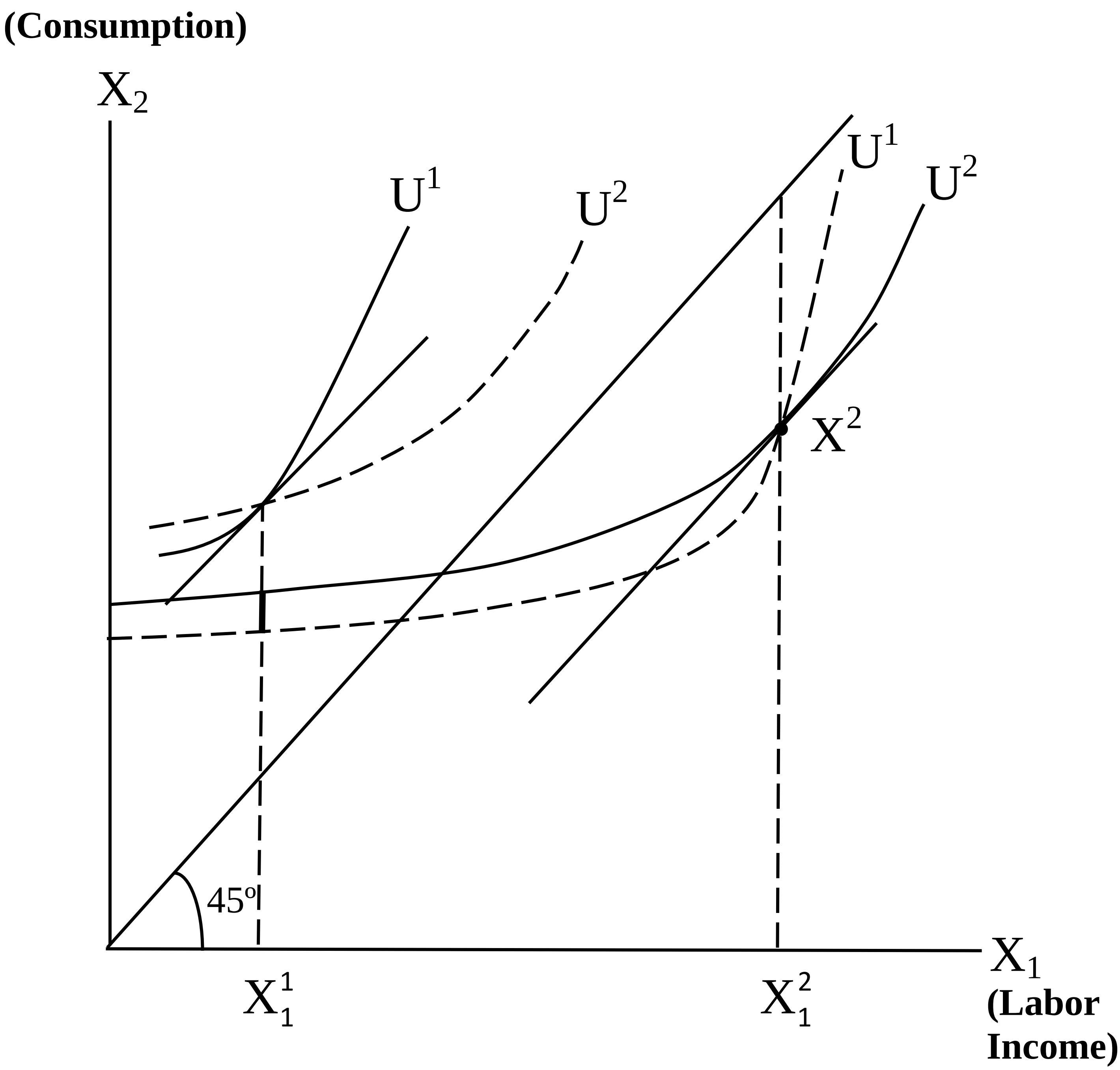

Figure 1

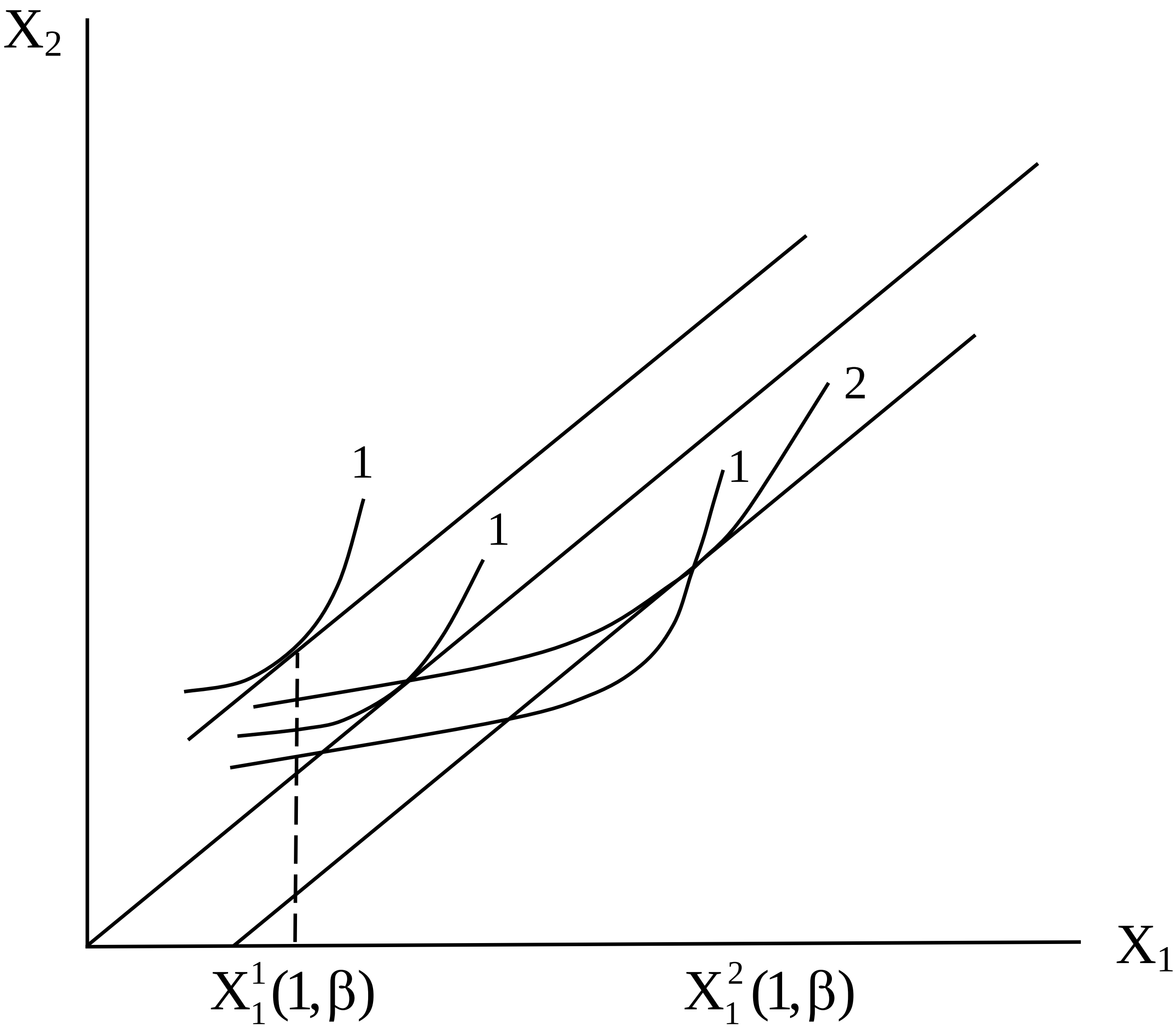

Figure 2

Appendix A (Proofs not in Main Text)

Proof of Lemma 1:

First, assume that $U^j\left(X_1^j, X_2^j\right) \leq U^j\left(X_1^k, X_1^k\right)$. Then assigning both individuals (X_1^k, X_1^k) would be Pareto superior to assigning the pair (X^1, X^2). To be allowable, this alternative assignment must be feasible and be a unique Nash equilibrium under some off-equilibrium consumptions. Since (X_1^k, X_1^k) is on the no-tax 45^o line, it is feasible. If one individual chooses some X_1 instead of X_1^k, set that individual's consumption at 0 and give the X_1 to the other individual. If both choose an income other than X_1^k, give each a level of consumption equal to the income chosen. This assignment always balances the budget and insures that each choosing X_1^k is the unique Nash equilibrium. That there is an allowable Pareto superior allocation contradicts (X^1, X^2) being a constrained efficient allocation so that $U^j\left(X_1^j, X_2^j\right) > U^j\left(X_1^k, X_1^k\right)$ must hold. Second, $U^j\left(X_1^j, X_2^j\right) > U^j\left(X_1^k, \hat{X}_2^k\right)$ follows from single crossing since $X_1^k > X_1^j$ if j = 1 and $X_1^k < X_1^j$ if j = 2.

Proof of Lemma 2:

(a) Setting $X(0) = \left(X_1^2, X_1^2\right)$ creates a bundle that balances the budget and lies below type 1's indifference curve through X^1 since $U^1(X^1) > U^1(X_1^2, X_1^2)$ from Lemma 1. Hence, with budget balance at X(0), type 1 is induced to choose X_1^1 when type 2 chooses X_1^2. In turn, $X(2)$ must be selected to induce type 1 to choose X_1^1 when type 2 chooses that income and to induce type 2 to choose X_1^2 when type 1 chooses X_1^1. Consider the indifference curves of the two types through the optimal bundle for type 2, X^2. On the vertical line through X_1^1, the indifference curve for type 1 will lie below that of type 2 because of single crossing. In case (i), if $X_2(2) = X_1^1$, then the budget is balanced at X(2). In addition in this case, X(2) lies between the indifference curves of the two individuals that go through X^2. Type 1 then chooses X(2) over X^2 and type 2 does the reverse. In (ii), the bundle ($X_1^1, X_2(2)$) would lie above the no tax 45° line for the smallest value of $X_2(2)$ that would make type 1 prefer X(2) to X^2. Thus, a deficit would be needed when both earn X_1^1 to ensure that type 1 would choose X_1^1 when type 2 chooses that income. In case (iii), the bundle $\left(X_1^1, X_2(2)\right)$) must lie below the no tax 45° line with a surplus required to ensure that type 2 would prefer X_1^2 when type 1 chooses X_1^1.

(b) If $U^2\left(X_1^2, X_1^2\right) \geq U^2\left(X_1^1, X_2^1\right)$, then $U^2(X^2) \geq U^2(X_1^1, X_1^1)$ follows from assumption (iv).[1] With X(0) and X(2) each set to satisfy budget balance, these conditions imply that type 2 has a dominant strategy to choose X_1^2. From Lemma 1, type 1 will then choose X_1^1 when type 2 chooses X_1^2 yielding the required dominance solvability. If $U^2(X_1^2, X_1^2) < U^2(X^1)$, then X(0) would need to be in deficit in order to ensure that type 2 chooses X_1^2 when type 1 chose X_1^1.

Proof of Lemma 3:

From Lemma 1, $U^2(X^2) > U^2(X_1^1, X_1^1)$. From the argument in footnote 1, $U^2(X_1^2, X_1^2) > U^2(X^1)$ must then hold so that type 2 has a dominant strategy. The condition in the Lemma ensures that if type 2 chooses X_1^2 then type 1 will choose X_1^1. When that condition is violated, X(0) must be

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[1] Assume that $U^i\left(a_1, a_2\right) = U^i\left(b_1, b_2\right)$ and $U^i\left(a_1, \bar{a}_2\right) = U^i\left(b_1, \bar{b}_2\right)$ where $a_1 < b_1$ and $a_2 < \bar{a}_2$. Then $\bar{b}_2 - b_2 \geq \bar{a}_2 - a_2$ must hold. If not, at some $a_1 < X_1 < b_1$, i's indifference curve through $\bar{a}_1$ would have to be flatter than i's indifference curve through a_1. At this value of X_1, leisure would be inferior for some values of p and K.

above the no-tax 45^o line in order to get type 1 to make the desired choice, and hence a deficit must occur at X(0). **QED**

Proof of Lemma 4:

If redistribution is from 2 to 1 and satisfies the **MS** constraints, then $U^2(X^2) \geq U^2(X^1)$. Since $U^2(X(0)) > U^2(X^2)$ (same earnings but X(0) has more consumption), $U^2(X(0)) > U^2(X^1)$. Since $U^2(X^1) > U^2(X(2))$ (same earnings but X(2) has less consumption), then $U^2(X^2) > U^2(X(2))$ follows. Hence, 2 has a dominant strategy to earn X_1^2. Since 1 is receiving a transfer, then, from Lemma 1, $U^1(X^1) > U^1(X(0))$, so 1 chooses X_1^1 when 2 chooses X_1^2. A symmetric argument holds when the transfer is from 1 to 2 with the type 1 having a dominant strategy. **QED**

Proof of Theorem 1:

From Lemma 3, the government can always do at least as well as with generalized taxes. The argument in the proof of that Lemma actually shows that whenever the **MS** self-selection constraints are satisfied even at distorted allocations, then all the dominance solvability conditions are satisfied with strict inequality. When the **MS** allocation is distorted, this allows the government to adjust the allocation to improve social welfare while satisfying all constraints.

Proof of Theorem 2:

(A) Let $X^1(\alpha^o)$ and $X^2(\alpha^o)$ be the optimal bundles on the no-tax 45^o line. From assumptions (iii) and (iv), as α increases, X_1^2 does not decrease and X_2^2 goes to 0. Hence, in the limit as α goes to 1, at least an amount arbitrarily close to $X_2^2(\alpha^o)$ will be extracted from the type 2 and transferred to the type 1 so that for sufficiently large α, $X_2^1(\alpha) > X_2^2(\alpha^o)$. In addition, $X_1^1(\alpha) \leq X_1^1(\alpha^o) < X_2^1(\alpha^o)$ from assumption (iv) and single crossing. Then,

$\lim_{\alpha \to 1} U^2\left(X^1(\alpha)\right) > U^2\left(X^2\left(\alpha^o\right)\right)$. Thus, for sufficiently large α, 2's indifference curve through $X^1(\alpha)$ must lie above $(X_1^2(\alpha), X_1^2(\alpha))$ and there must be a deficit at X(0) for 2 to have a dominant strategy.

(B) As α rises, $U^2(\alpha) \equiv U^2(X^2(1,\alpha))$ declines. For any X_1^2, denote as $X_2^2(X_1^2, U^2(\alpha))$ the consumption X_2^2 that is needed to reach $U^2(\alpha)$. Since this consumption level declines to zero as α increases, for sufficiently large α, $X_2^2(\hat{X}_1^1, U^2(\alpha)) < \hat{X}_1^1$ must hold. Then, for all X_1^1, with $\hat{X}_1^1 < X_1^1 < X_1^2(1,\alpha)$ (which includes $X_1^1 = X_1^1(1,\alpha)$), $X_2^2(X_1^1, U^2(\alpha)) < X_1^1$ must hold. Hence, a surplus is needed to sustain the full-information allocation.

(C) To sustain $X^1(1,\alpha)$ and $X^2(1,\alpha)$ when $X_1^1(1,\alpha) = 0$ requires $X_1(2) = 0$ and $U^1(X(2)) > U^1(X^2(1,\alpha))$. From the assumptions on preferences, $X_2(2) > 0$, thus a deficit occurs when both report they are type 1. **QED**

Proof of Theorem 3:

(a) If an off-equilibrium-path deficit is needed to sustain the full-information allocation and type 1 has a dominant strategy to earn $X_1^1(1,\alpha)$, then the government's optimization problem reduces to:

$$\max_{X_1^1, X_2^1, X_1^2, X_2^2} \quad \alpha U^1(X_1^1, X_2^1) + (1-\alpha) U^2(X_1^2, X_2^2)$$

$$\text{s.t.} \quad X_1^1 - X_2^1 + X_1^2 - X_2^2 \geq 0 : \mu$$

$$U^1(X_1^1, X_1^1) \geq U^1(X_1^2, X_2^2) : \lambda$$

where the binding incentive constraint is that type 1 prefers the bundle intended for him over that for type 2 when type 2 has selected type 1 income. The first-order conditions reduce to:

$$\alpha[U_1^1(X_1^1,X_2^1)+U_2^1(X_1^1,X_2^1)]=-\lambda[U_1^1(X_1^1,X_1^1)+U_2^1(X_1^1,X_1^1)] \quad \text{(A1)}$$

$$(1-\alpha)[U_1^2(X_1^2,X_2^2)+U_2^2(X_1^2,X_2^2)]=\lambda[U_1^1(X_1^2,X_2^2)+U_2^1(X_1^2,X_2^2)] \quad \text{(A2)}$$

$$\alpha U_2^1(X_1^1,X_2^1)-(1-\alpha)U_2^2(X_1^2,X_2^2)=-\lambda U_2^1(X_1^2,X_2^2) \quad \text{(A3)}$$

From (A1) and normality, $U_1^1(X_1^1,X_2^1)+U_2^1(X_1^1,X_2^1)<0<U_1^1(X_1^1,X_1^1)+U_2^1(X_1^1,X_1^1)$. Hence, $MRS^1(X_1^1,X_2^1)>1$, so that X_1^1 is greater than the undistorted income on that $p\cdot X=K$ line. Second, from (A2), $1-MRS^1(X_1^2,X_2^2)$ and $1-MRS^2(X_1^2,X_2^2)$ must be of the same sign. From a bundle such that $1>MRS^1(X_1^2,X_2^2)>MRS^2(X_1^2,X_2^2)$, the allocation is inferior to one with a higher level for X_1^2 on the same $p\cdot X=K'$ budget line. Both types' indifference curves through (X_1^2,X_2^2) must cross the $p\cdot X=K'$ budget line a second time at higher incomes with the intersection for type 2 at a higher income than the intersection for type 1. Any income on the $p\cdot X=K'$ line lying between those two intersections raises 2's utility and relaxes the incentive constraint on 1. See Figure A1. Thus, an allocation with $1>MRS^1(X_1^2,X_2^2)>MRS^2(X_1^2,X_2^2)$ is subject to a Pareto improvement. Hence, $MRS^1(X_1^2,X_2^2)>MRS^2(X_1^2,X_2^2)>1$ must hold. See Figure A2.

From (A3) if X_1^1 and X_1^2 were fixed and the incentive constraint removed, X_2^1 would be reduced and X_2^2 increased since $\alpha U_2^1(X_1^1,X_2^1)<(1-\alpha)U_2^2(X_1^2,X_2^2)$.

(b) In this case, the binding incentive constraint is $U^2(X_1^2,X_1^2)\geq U^2(X_1^1,X_2^1)$. The first-order conditions reduce to:

$$\alpha[U_1^1(X_1^1,X_2^1)+U_2^1(X_1^1,X_2^1)]=\lambda[U_1^2(X_1^1,X_2^1)+U_2^2(X_1^1,X_2^1)] \quad \text{(A4)}$$

$$(1-\alpha)[U_1^2(X_1^2,X_2^2)+U_2^2(X_1^2,X_2^2)]=-\lambda[U_1^2(X_1^2,X_1^2)+U_2^2(X_1^2,X_1^2)] \quad \text{(A5)}$$

$$\alpha U_2^1(X_1^1, X_2^1) - (1-\alpha)U_2^2(X_1^2, X_2^2) = \lambda U_2^2(X_1^2, X_2^2) \tag{A6}$$

From the incentive constraint, $U_1^2(X_1^1, X_2^1) + U_2^2(X_1^1, X_2^1) > 0$, so (A4) requires $U_1^1(X_1^1, X_2^1) +$ $U_2^1(X_1^1, X_2^1) > 0$ or $MRS^1(X_1^1, X_2^1) < 1$. From normality, $MRS^2(X_1^2, X_1^2) >$ $MRS^2(X_1^2, X_2^2)$. From (A5), $1 - MRS^2(X_1^2, X_2^2)$ and $1 - MRS^2(X_1^2, X_1^2)$ must have opposite signs, so $MRS^2(X_1^2, X_2^2) < 1$ must hold. Since $\alpha U_2^1(X_1^1, X_2^1) > (1-\alpha)U_2^2(X_1^2, X_2^2)$, the transfer would be increased if the incentive constraint were removed at these income levels.

(2) From Lemma 3, if redistribution is toward the type 2, then type 2 must have the dominant strategy and the no-deficit constraint can only bind with respect to type 1 at X(0). The government's optimization then reduces to:

$$\max_{X_1^1, X_2^1, X_1^2, X_2^2} \quad \alpha U^1(X_1^1, X_2^1) + (1-\alpha)U^2(X_1^2, X_2^2)$$

$$\text{s.t.} \quad X_1^1 - X_2^1 + X_1^2 - X_2^2 \geq 0 : \mu$$

$$U^1(X_1^1, X_2^1) \geq U^1(X_1^2, X_1^2) : \lambda$$

The first order conditions reduce to:

$$(\alpha + \lambda)[U_1^1(X_1^1, X_2^1) + U_2^1(X_1^1, X_2^1)] = 0 \tag{A7}$$

$$(1-\alpha)[U_1^2(X_1^2, X_2^2) + U_2^2(X_1^2, X_2^2)] = \lambda[U_1^1(X_1^2, X_1^2) + U_2^1(X_1^2, X_1^2)] \tag{A8}$$

$$\alpha U_2^1(X_1^1, X_2^1) - (1-\alpha)U_2^2(X_1^2, X_2^2) = -\lambda U_2^1(X_1^2, X_2^2) \tag{A9}$$

That $MRS^1\left(X_1^1, X_2^1\right) = 1$ follows immediately from (A7). Since (X_1^1, X_2^1) and (X_1^2, X_1^2) lie on the same indifference curve for type 1 given the binding no-deficit constraint and (X_1^1, X_1^1) is at a lower income level and on an inferior 45 line, then $MRS^1\left(X_1^2, X_1^2\right)$ must be greater than 1. From

(A8), $1\text{-MRS}^2\left(X_1^2, X_2^2\right)$ and $1\text{-MRS}^1\left(X_1^2, X_1^2\right)$ must have the same sign, so $\text{MRS}^2\left(X_1^2, X_2^2\right) > 1$ follows immediately. That there is excess transfer follows immediately from (A9). **QED**

Proof of Theorem 4: (1) Assume that $n^1 = n^2 = 1$ and let $\bar{\alpha}$ be the social weight such that the maximum redistribution is being done toward type 1s without the need to create any distortions. Denote $X^i\left(1, \bar{\alpha}\right)$ as the undistorted bundles that would solve problem **(II)** for α equal to $\bar{\alpha}$. Then the bundles $X^i(1) = X^i\left(1, \bar{\alpha}\right)$, $X(2) = \left(X_1^1\left(1, \bar{\alpha}\right), X_1^1\left(1, \bar{\alpha}\right)\right)$, and $X(0) = \left(X_1^2\left(1, \bar{\alpha}\right), X_1^2\left(1, \bar{\alpha}\right)\right)$ balance the budget for all states. They satisfy dominance solvability and yield the maximum undistorted redistribution if $U^2\left(X^2\left(1, \bar{\alpha}\right)\right) = U^2\left(X(2)\right)$. See Figure A3.

Now assume that $n^1 = n^2 = 2$. Assigning $X^i\left(1, \bar{\alpha}\right)$ to each individual of type i will still be a full-information Pareto optimum in the larger economy. If it is to be implemented, then $X^2(2)$ must equal $X^1\left(1, \bar{\alpha}\right)$. Consider the bundle X(4) that is assigned when everyone claims to be type 1. With action revelation, $X_1(4)$ must equal $X_1^1\left(1, \bar{\alpha}\right)$ and then due to budget balance, $X_2(4)$ must also equal $X_1^1\left(1, \bar{\alpha}\right)$. Hence, X(4) must be the same as the bundle X(2) when there were only two individuals.

Consider type 1's indifference curve through the bundle X(4). First, assume that this intersects the line $X_2^2 = X_2^2\left(1, \bar{\alpha}\right)$ below the $p \cdot X = 0$ line. Denote the intersection as point A. Since the dominance solvability constraints imply that $U^1\left(X(4)\right) \geq U^1\left(X^2(3)\right)$ and that $U^2\left(X^2(3)\right) \geq U^2\left(X(4)\right)$, then $X^2(3)$ must lie on the line segment between point A and $X^2\left(1, \bar{\alpha}\right)$. To balance the budget when $N^1 = 3$, $X^1(3)$ must be above $X^1(4)$ on the line

$X_1^1 = X_1^1(1,\ \bar{\alpha})$. But then $U^2\left(X^2(3)\right) > U^2\left(X(4)\right) = U^2\left(X^2(1,\ \bar{\alpha})\right)$, which violates the incentive constraint that $U^2\left(X^2(2)\right) \geq U^2\left(X^1(3)\right)$. Hence, in this situation, the bundles $X^2(1,\ \bar{\alpha})$ cannot be implemented when there are two individuals of each type.

A second possibility is if type 1's indifference curve through the bundle X(4) intersects the line $X_2^2 = X_2^2(1,\ \bar{\alpha})$ above the $p \cdot X = 0$ line. Since the $X^2(3)$ bundle can now be chosen so that $p \cdot X^2(3) > 0$, then the $X^1(3)$ bundle will be below X (4). This implies that $U^2\left(X^2(1,\ \bar{\alpha})\right) > U^2\left(X^1(3)\right)$. Not only is the allocation $X^i(1,\ \bar{\alpha})$ sustainable, but because the dominance solvability constraints all hold with strict inequality, it is actually possible to sustain allocations $X^i(1,\ \alpha)$ for at least some $\alpha > \bar{\alpha}$. In this case, more undistorted allocations are implementable with four individuals than with two. **QED**

Proof of Theorem 5: If an off-equilibrium-path surplus is needed to sustain the full-information allocation, from Lemmas 2 and 3, this can only occur when transfers are from type 2 to type 1 and type 1 has a dominant strategy. The binding incentive constraint is that type 2 prefers to earn the income intended for him when type 1 has chosen the appropriate income and the optimization problem becomes:

$$\max_{X_1^1, X_2^1, X_1^2, X_2^2} \alpha U^1(X_1^1, X_2^1) + (1-\alpha)U^2(X_1^2, X_2^2)$$

$$\text{s.t.} \quad X_1^1 - X_2^1 + X_1^2 - X_2^2 \geq 0 \ : \ \mu$$

$$U^2(X_1^2, X_2^2) \geq U^2(X_1^1, X_1^1) \ : \ \lambda$$

The first-order conditions reduce to:

$$\alpha[U_1^1(X_1^1, X_2^1) + U_2^1(X_1^1, X_2^1)] = \lambda[U_1^1(X_1^1, X_1^1) + U_2^1(X_1^1, X_1^1)] \qquad \text{(A10)}$$

$$[1 - \alpha + \lambda][U_1^2(X_1^2, X_2^2) + U_2^2(X_1^2, X_2^2)] = 0 \qquad \text{(A11)}$$

$$\alpha U_2^1(X_1^1, X_2^1) - (1-\alpha)U_2^2(X_1^2, X_2^2) = \lambda U_2^2(X_1^2, X_2^2) \quad \text{(A12)}$$

Since $U_1^1(X_1^1, X_1^1) + U_2^1(X_1^1, X_1^1) > 0$ follows as in the proof of Theorem 2 and $\lambda U_2^2 > 0,$ the result here follows immediately. **QED**

Appendix B (on renegotiation proofness)

Consider the case of just one individual of each type where the government wants to sustain a pair of bundles Z^1 and Z^2 with $Z_1^1 < Z_1^2$ and $Z_2^1 + Z_2^2 = Z_1^1 + Z_1^2$. It must announce tax schedules that imply consumptions for each individual for any pair of incomes $X_1 = (X_1^a, X_1^b)$. Denote the consumptions implied by these tax schedules as $C^a\left(X_1^a, X_1^b\right)$ and $C^b\left(X_1^a, X_1^b\right)$. Assume that these satisfy the following properties:

$$C^a\left(X_1^a, X_1^b\right) + C^b\left(X_1^a, X_1^b\right) = X_1^a + X_1^b, \text{ any } X_1^a \text{ and } X_1^b \quad \text{(B1)}$$

$$C^a(Z_1^1, Z_1^2) = Z_2^1 \quad \text{(B2)}$$

$$C^a(X_1^a, X_1^b) = C^b(X_1^b, X_1^a), \text{ for all } X_1^a \text{ and } X_1^b \quad \text{(B3)}$$

$$C^a(X_1^i, Z_1^2) = C^a(Z_1^1, X_1^i) = 0, \text{ for any } Z_1^1 \neq X_1^i \neq Z_1^2 \quad \text{(B4)}$$

$$C^i(X_1^a, X_1^b) = X_1^i, \ i = a, b, \text{ any } X_1^a \text{ and } X_1^b \text{ with } Z_1^1 \neq X_1^i \neq Z_1^2, \ i = a, b \quad \text{(B5)}$$

Condition (B1) imposes budget balance for all X_1 and thus ensures that the schedules are renegotiation proof. Condition (B2) asserts that if individuals choose the two desired incomes, then they receive the appropriate consumptions. This is clearly necessary if the bundles Z^1 and Z^2 are to be sustained. Condition (B3) says that the taxes depend on the pair of incomes earned and not on the name of the individuals. Conditions (B1) and (B3) together imply that if the individuals pool and choose the same income then each pays no tax with consumption equal to income. Condition (B4) says that if one individual chooses one of the two desired incomes and the other does not, then the one who did not is punished with zero consumption and, from (B1), the one who chooses a desired income is given a bonus. This imposes the maximum penalty possible on an individual who does not select a desired income when the other individual does. Condition (B5) says that if neither chooses a desired income then neither pays any tax nor receives any subsidy. Given budget balance, to punish one of them would require rewarding the other who has also not chosen a desired income. There may be specific circumstances in which rewarding one and punishing the other will help to eliminate selecting incomes other than Z_1^1 and Z_1^2 but there does not seem to be a general rule as to who is to be rewarded and who punished.

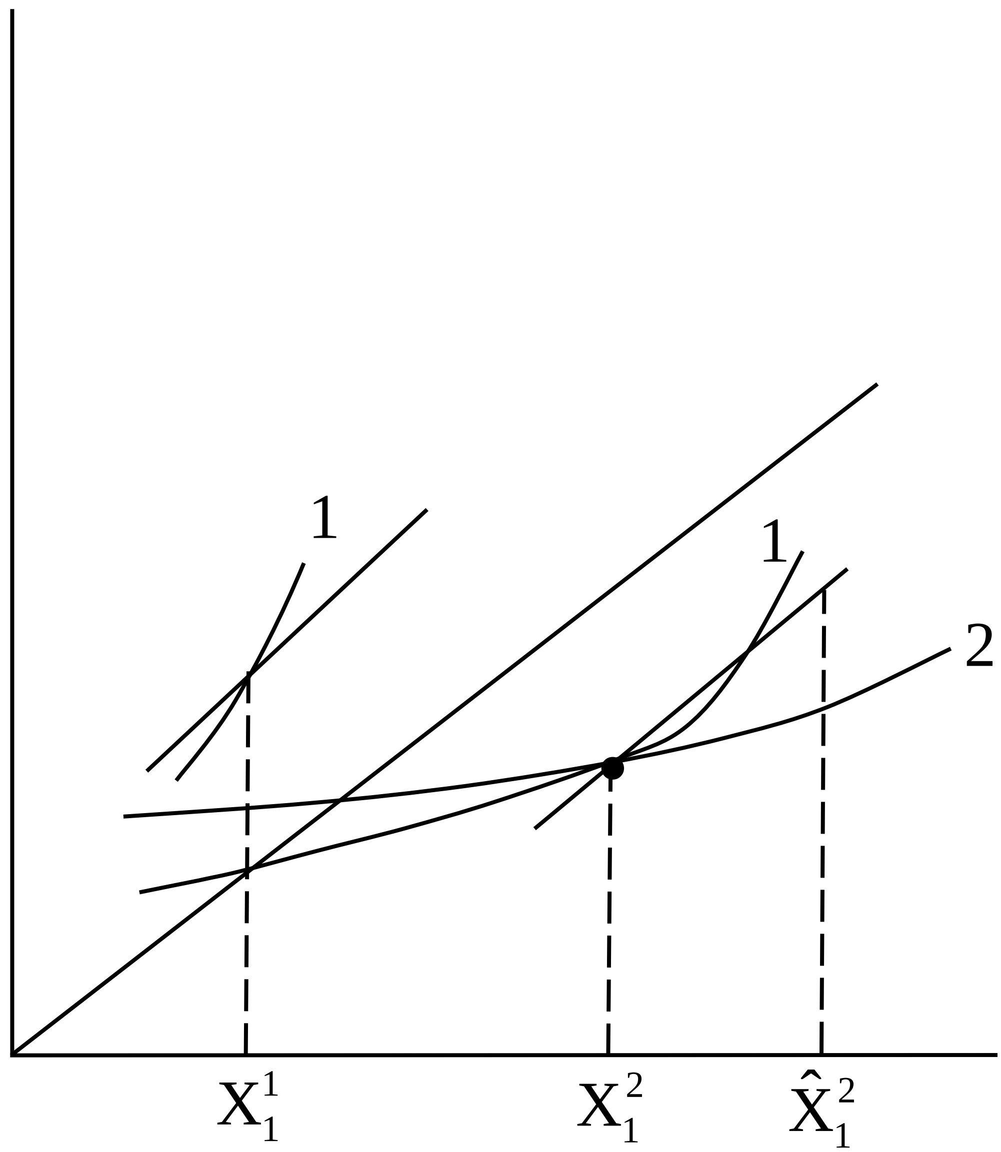
Lemma 6: Consider an economy with one individual of each type where the government wishes to sustain bundles Z^1 and Z^2 as an equilibrium. Facing the tax schedules defined by (B1) – (B5), the only possible Nash equilibrium incomes (X_1^a, X_1^b) in the game in stage 3 are $\{(Z_1^1, Z_1^1), (Z_1^2, Z_1^2), (Z_1^1, Z_1^2), (Z_1^2, Z_1^1)\}$ provided that $Y_1^i \geq Z_2^k - Z_1^k$ where i is the individual paying the net tax.

Proof of Lemma 6: Consider any pair of incomes not among the four given. There are two possibilities: (a) one individual selects an income from Z_1^1 and Z_1^2 while the other does not or (b) neither selects an income from Z_1^1 and Z_1^2. The situation in (a) cannot be a Nash equilibrium since the individual who did not select an income from Z_1^1 and Z_1^2 would receive zero consumption given (B4) and, given assumptions (i) – (iii) on preferences, would gain by unilaterally choosing the same income as the other individual and receiving consumption Z_1^k from (B1) and (B3). In situation (b) the only possible Nash equilibrium would be for both individuals to select their best points Y_1^i on the $p \cdot X^i = 0$ line. An individual who was not at such a point would gain by moving there whether or not that Y_1^i was one of the Z_1^k. The individual would be at a better point on the $p \cdot X^i = 0$ line if it were not and would be given an additional transfer from the other individual if it were given (B1) and (B4). Denote by k the individual who would be receiving a transfer in the allocation that the government wishes to sustain. Then $U^k(Z^k) > U^k(Y_1^i, Y_1^i)$. If not, then assigning Y^1 and Y^2 would be a feasible Pareto improvement. Since $Z_1^k + Y_1^i \geq Z_2^k$ from the condition in the Lemma, then $U^k\left(Z_1^k, Z_1^k + Y_1^i\right) \geq U^k\left(Z^k\right) > U^k\left(Y_1^k, Y_1^k\right)$, so k would not remain at Y^k. Thus, all allocations other than the four in the Lemma are ruled out as Nash equilibria.

QED

The additional condition does limit the amount of redistribution done by the government since it requires that the transfer to the individual who gains relative to the no tax situation is less than the other individual's optimal no tax income. If the government wished to do more redistribution than that, it might be possible with these tax schedules since the condition is sufficient but not necessary or by using different tax schedules that modify condition (B5). Given the result in Lemma 6, incomes other than the desired ones are ruled out in equilibrium. Dominance solvability conditions over just the two desired incomes can then be imposed as in the full-commitment case to ensure that a unique equilibrium exists in the stage 3 game among individuals.

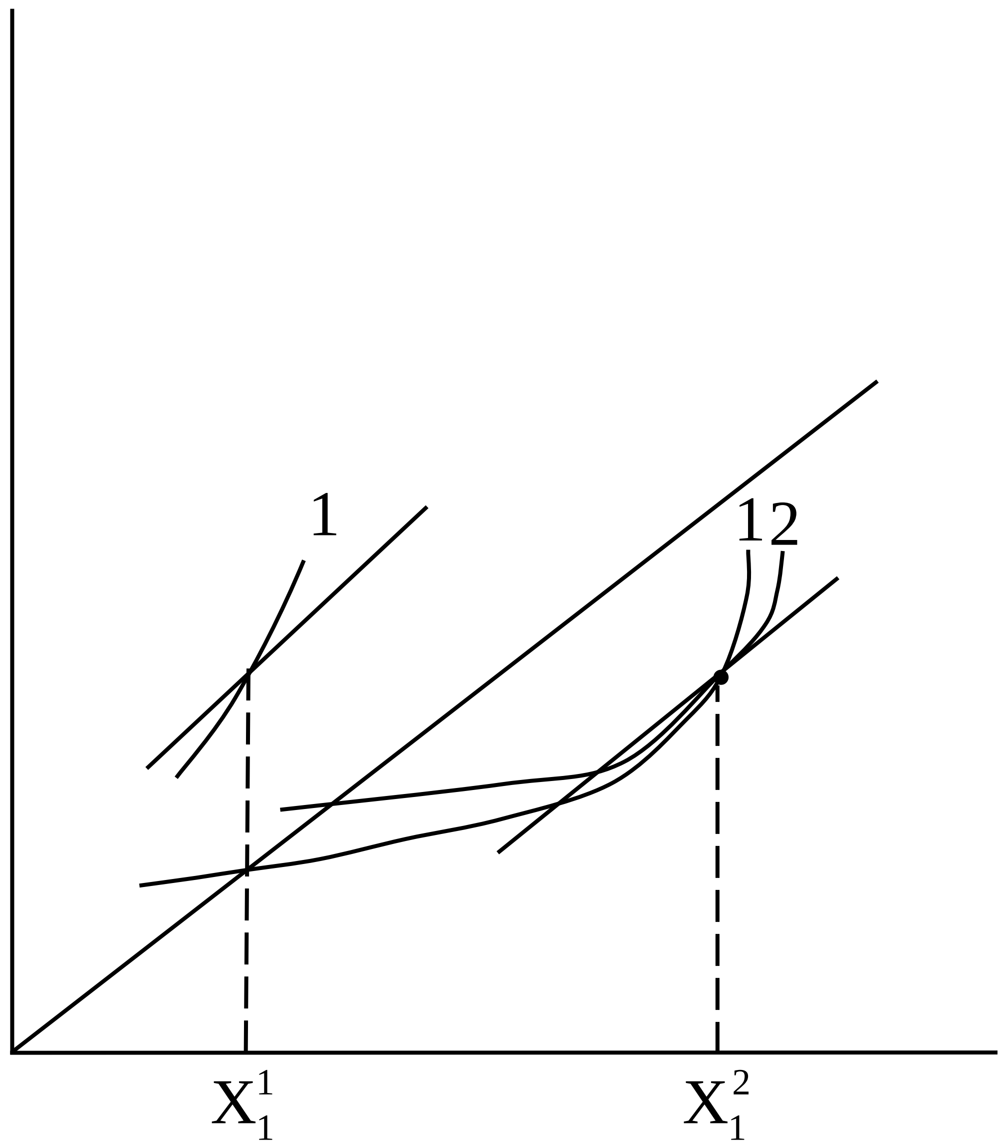
The argument behind Lemma 6 is even stronger when there are more than two individuals. With a straightforward modification of conditions (B1) to (B5), it follows, just as in the proof above, that the only possible Nash equilibrium other than every individual choosing some income from the set the government is trying to sustain is for everyone to choose their optimal income on the no-tax 45^{o} line. Ruling that out as a Nash equilibrium requires a weaker condition than that in the Lemma since one person deviating to a desired income would receive not just one other person's income but the total income earned by everyone else.



$$1 > MRS^1(X_1^2, X_2^2) > MRS^2\left(X_1^2, X_2^2\right)$$

Increasing type 2's earnings to $\hat{X}_1^2$ with the same total tax results in a Pareto improvement.

Figure A1



$$MRS^1(X_1^2, X_2^2) > MRS^2\left(X_1^2, X_2^2\right) > 1$$

Figure A2

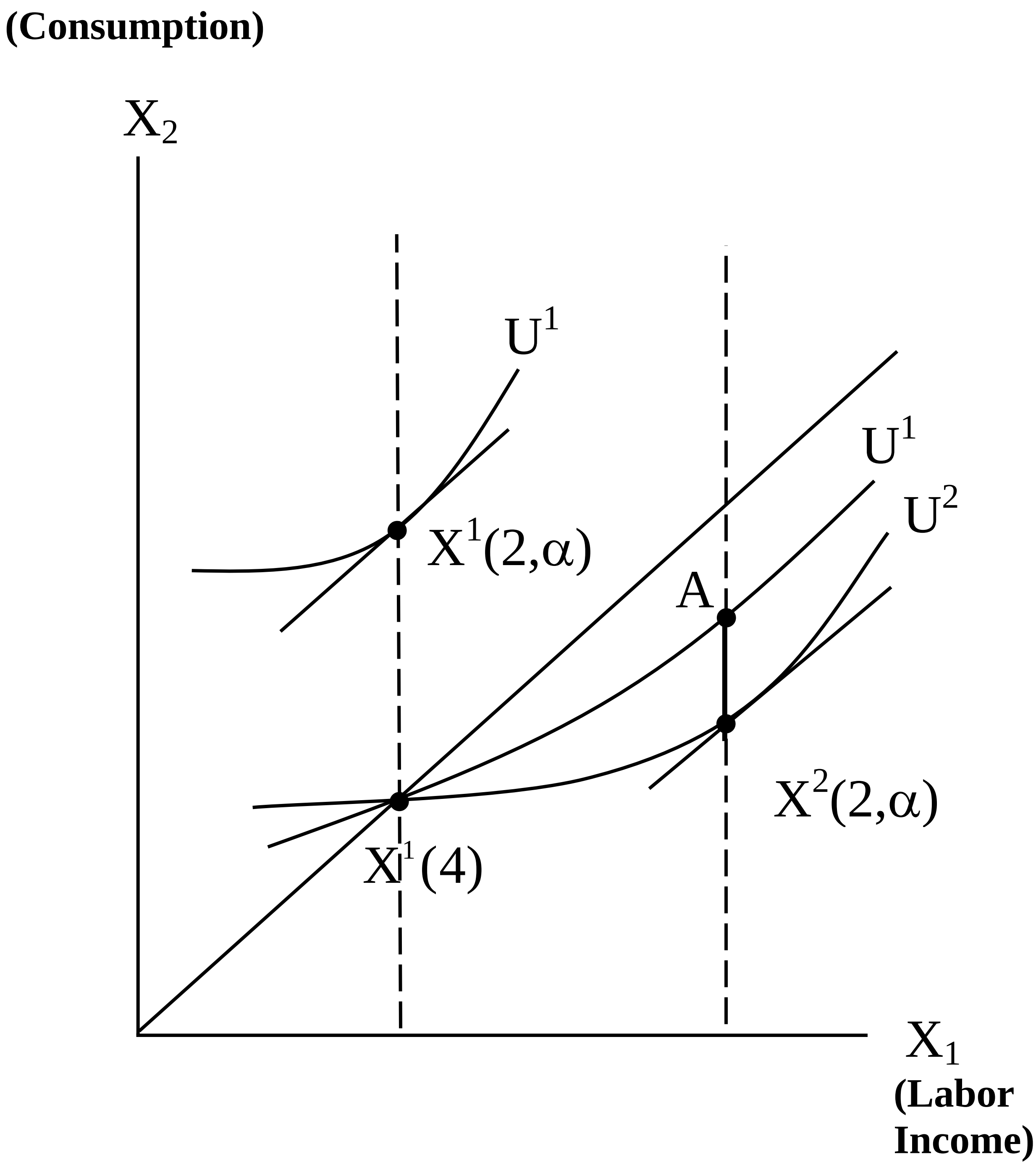


Figure A3