Directions: Answer each question. Simplify. (10 points)

1. (5 points) Find parametric equations for the tangent line to the curve at the point \((8, \ln 64, 1)\) for the equation

\[ x = \sqrt{t^2 + 63}, \quad y = \ln (t^2 + 63), \quad z = t \]

_Solution:_ Setting the \(z\) component of the equation equal to the \(z\) component of the point \((8, \ln 64, 1)\), gives that \(t = 1\). The derivative \(r'(t) = \left< \frac{t}{\sqrt{t^2 + 63}}, \frac{2t}{t^2 + 63}, 1 \right>\) gives that \(r'(1) = \left< \frac{1}{8}, \frac{1}{32}, 1 \right>\). So the answer is \(\left< \frac{t}{8} + 8, \frac{t}{32} + \ln (64), t + 1 \right>\).

2. (5 points) Find the length of the curve where \(0 \leq t \leq \frac{\pi}{4}\)

\[ r(t) = \langle \cos (8t), \sin (8t), 8 \ln (\cos (t)) \rangle \]

_Solution:_

\[
\int_0^{\pi/4} \sqrt{(-8 \cos (8t))^2 + (8 \sin (8t))^2 + \left(8 \frac{\sin (t)}{\cos (t)}\right)^2} \, dt = \int_0^{\pi/4} \sqrt{64 (1) + 64 \tan^2 (t)} \, dt
\]

\[
= \int_0^{\pi/4} 8 \sqrt{\sec^2 t} \, dt = \int_0^{\pi/4} 8 \sec t \, dt = 8 \ln (\sqrt{2} + 1)
\]