MHF 3202 (Larson) Final Forms of Proofs

1. Final form of a proof of $(\forall x)(P(x))$:

   *Proof:* Let $x$ be arbitrary. (o$\forall$)

   [Proof of $P(x)$]

   Since $x$ was arbitrary, true for all such and $(\forall x)(P(x))$ follows. (c$\forall$) $\square$

2. Final form of a direct proof of $P \to Q$:

   *Proof:* Assume $P$ (o$\to$)

   [Proof of $Q$]

   We assumed $P$ and proved $Q$ so the implication follows. (c$\to$) $\square$

3. Final form of a proof by contrapositive of $P \to Q$:

   *Proof:* Toward the contrapositive, assume $\neg Q$ (o$\to$)

   [Proof of $\neg P$]

   We assumed $\neg Q$ and proved $\neg P$ so the implication follows. (c$\to$) Therefore by the contrapositive, $P$ implies $Q$ is true. $\square$

4. Final form of a proof of $P \to Q$ by contradiction:

   *Proof:* Assume toward a contradiction that $P \to Q$ is false. (o *) That is, assume $P$ is true and $Q$ is false.

   [Proof of a contradiction, such as proving $\neg P$ and getting the contradiction $P \wedge \neg P$]

   We assumed $P \to Q$ was false and reached a contradition, so our assumption was false and the original implication follows. (c *) $\square$

5. Final form of a proof of $P \vee Q$ (informal cases):

   *Proof:* If $P$ is true, then of course $P \vee Q$ is true (case 1). So assume $\neg P$ is true.

   [Proof of $Q$]

   Since $Q$ is true, it follows that $P \vee Q$ is true (case 2). Thus $P \vee Q$ is true since it is true whether $P$ is true or false. (a cases)

6. Final form of a proof of $(\exists x)(P(x))$:

   *Proof:* Let $x_0 =$ [the value you decided on].

   [Proof of $P(x)$]

   Thus $(\exists x)(P(x))$ since $x_0$ is a witness. (p$\exists$) $\square$

7. Final form of a proof of $P \wedge Q$:

   *Proof:*

   [Proof of $P$]

   [Proof of $Q$]

   Thus $(P \wedge Q)$ since both $P$ and $Q$ are true. (p$\wedge$) $\square$

8. Final form of a proof of $P \leftrightarrow Q$:

   *Proof:*

   [Proof of $P \to Q$]

   [Proof of $Q \to P$]

   Thus $(P \leftrightarrow Q)$ since both $P \to Q$ and $Q \to P$ are true. (p$\leftrightarrow$) $\Box$

Additional Proof Strategies with Annotations

1. Use of $\neg P$:

   We know $\neg P$.

   [re-expression of $\neg P$ as a positive statement $Q$] (re-exp)

2. Use of $P \vee Q$ when we know or can prove $\neg Q$:

   We know $P \vee Q$ and $\neg Q$, so $P$ hold by disjunctive syllogism.(ds)

3. Use of knowing that one of $H_1, H_2, \ldots, H_n$ is true to prove $R$:

   Since we know one of $H_1, H_2, \ldots, H_n$ is true, we use cases to prove $R$.

   Case 1: $H_1$ is true
   [Proof of R]

   Case 2: $H_2$ is true
   [Proof of R]

   $\vdots$

   Case n: $H_n$ is true
   [Proof of R]

   Thus by exhaustive case analysis, $R$ is true. (a cases)

4. Use of $(\exists x)(P(x))$:

   We know $(\exists x)(P(x))$. Let $x_0$ be a witness; that is assume $P(x_0)$. (a$\exists$)

5. Use of $(\forall x)(P(x))$:

   We know $(\forall x)(P(x))$.

   [Let $a$ be a value for which you decide it is useful to know $P(a)$]

   Since $(\forall x)(P(x))$, we know that $P(a)$. (a$\forall$)

6. Use of Modus Ponens, $P$ and $P \to Q$:

   Since $P$ and $P \to Q$, it follows that $Q$ holds. (mp)

7. Use of $P \wedge Q$.

   Since $P \wedge Q$ we know both $P$ and $Q$ are true. (a$\wedge$)

8. use of $P \leftrightarrow Q$.

   Since $P \leftrightarrow Q$, we know that $P \to Q$ and $Q \to P$ are both true. (definition of $\leftrightarrow$ or (a$\leftrightarrow$)).

9. Use of Modus Tollens, $\neg Q$, and $P \to Q$:

   Since $\neg Q$ and $P \to Q$, it follows that $\neg P$ holds. (mt)