let \( x(t) \): = displacement to right of \( m_1 \) block

\( y(t) \): = displacement to right of \( m_2 \) block

The block with mass \( m_1 \) has the force

\[
F_1 = -k_1x + k_1y = k_1(y-x)
\]

acting on it and the block with mass \( m_2 \) has the forces

\[
F_2 = -k_2y
\]

\[
F_3 = k_1x - k_1y = k_1(x-y)
\]

So we have the system of differential equations

\[
m_1 \cdot x''(t) = F_1 = k_1(y-x)
\]

\[
m_2 \cdot y''(t) = F_2 + F_3 = -k_2y + k_1(x-y)
\]

Substituting \( m_1 = 1k \), \( m_2 = 2k \), \( k_1 = 4 \frac{N}{m} \), \( k_2 = \frac{10}{3} \frac{N}{m} \) we get

\[
D^2x = 4y - 4x
\]

\[
2D^2y = -\frac{10}{3}y + 4x - 4y = 4x - \frac{22}{3}y
\]

or equivalently,
\[(D^2 + 4)x - 4y = 0\] 

\[-2x + (D^2 + \frac{11}{3})y = 0\]

Multiplying \(\Box\) by \((D^2 + \frac{11}{3})\) and \(\bigstar\) by 4 then adding the resulting equations yields

\[(D^2 + 4)(D^2 + \frac{11}{3})x = 8x = 0\]

\[\iff (D^4 + \frac{23}{3}D^2 + \frac{20}{3})x = 0\]

\[\iff (D^2 + 1)(D^2 + \frac{20}{3})x = 0\]

Hence, the roots of the associated auxiliary equation are

\[r = \pm i, \pm i\sqrt{\frac{20}{3}}\]

So the solution for \(x\) is given by

\[x(t) = C_1 \cos t + C_2 \cos \left( t \sqrt{\frac{20}{3}} \right)\]

(which is the sum of two linearly independent homogeneous solutions).
Now since

\[ D^2 \chi(t) = D^2 (c_1 \cos t + c_2 \cos (\sqrt{\frac{10}{3}} t)) \]

\[ = -c_1 \cos t - \frac{20}{3} c_2 \cos (\sqrt{\frac{10}{3}} t) \]

we have that (by substituting into \( \Phi \)):

\[ y(t) = \frac{1}{4} \left( D^2 + 4 \right) \chi(t) \]

\[ = \frac{1}{4} D^2 \chi(t) + \chi(t) \]

\[ = -\frac{1}{4} c_1 \cos t - \frac{5}{3} c_2 \cos (\sqrt{\frac{10}{3}} t) + c_1 \cos t + c_2 \cos (\sqrt{\frac{10}{3}} t) \]

\[ \Rightarrow \quad y(t) = \frac{3}{4} c_1 \cos t - \frac{2}{3} c_2 \cos (\sqrt{\frac{10}{3}} t) \]

Now it simply remains to solve for \( c_1 \) \& \( c_2 \) using the initial conditions:

\[ \chi(0) = -1 \quad \chi'(0) = 0 \]

\[ y(0) = 0 \quad y'(0) = 0 \]
Accordingly, we have

\[ -1 = X(0) = C_1 + C_2 \quad \text{①} \]

\[ 0 = Y(0) = \frac{3}{4} C_1 - \frac{2}{3} C_2 \quad \text{②} \]

From ② we have that \( C_1 = \frac{8}{9} C_2 \) which, when substituted into ①, yields

\[ -1 = \frac{8}{9} C_2 + C_2 \]

\[ \iff -1 = \frac{17}{9} C_2 \]

\[ \iff C_2 = -\frac{9}{17} \]

Then \( C_1 = \frac{8}{9} \left( -\frac{9}{17} \right) = -\frac{8}{17} \). Thus, our final solution to the system of differential equations is given by

\[
X(t) = -\frac{8}{17} \cos t - \frac{9}{17} \cos \left( \frac{12}{5} t \right)
\]

\[
Y(t) = -\frac{6}{17} \cos t + \frac{6}{17} \cos \left( \frac{10}{13} t \right)
\]