

Show your work to earn full credit.

1. Find the vector that has the same direction as $w = (3, 6, 2)$ but has length 3. (2 points)

Normalize w :
$$u = \frac{w}{|w|} = \frac{1}{\sqrt{3^2 + 6^2 + 2^2}} w = \frac{1}{\sqrt{49}} w = \frac{1}{7} w.$$

So $|u| = 1$. Hence, the vector with length 3 in direction of w is

$$3u = \frac{3}{7} \langle 3, 6, 2 \rangle = \boxed{\langle \frac{9}{7}, \frac{18}{7}, \frac{6}{7} \rangle}$$

2. Find the angle between $u = (2, 0, 1)$ and $v = (4, 2, 0)$ (leave your answer in the exact expression). (2 points)

$$\begin{aligned} u \cdot v &= 2 \cdot 4 + 0 \cdot 2 + 1 \cdot 0 = 8 \\ |u| &= \sqrt{2^2 + 1^2} = \sqrt{5} \\ |v| &= \sqrt{4^2 + 2^2} = \sqrt{20} = 2\sqrt{5} \end{aligned} \quad \left| \begin{aligned} \text{Since } u \cdot v &= |u| |v| \cos \theta \\ \Rightarrow \theta &= \cos^{-1} \left(\frac{u \cdot v}{|u| |v|} \right) \\ &= \cos^{-1} \left(\frac{8}{\sqrt{5} \cdot 2\sqrt{5}} \right) \\ &= \boxed{\cos^{-1} \left(\frac{4}{5} \right)} \end{aligned} \right.$$

3. Consider the points $P(3, 0, 3)$, $Q(2, 1, 5)$, $R(6, 2, 7)$.

- (i) Find a nonzero vector orthogonal to the plane through the points P , Q , and R . (2 points)

$$\vec{PQ} = \langle 2-3, 1-0, 5-3 \rangle = \langle -1, 1, 2 \rangle, \quad \vec{PR} = \langle 6-3, 2-0, 7-3 \rangle = \langle 3, 2, 4 \rangle$$

$$\text{Then } \vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ -1 & 1 & 2 \\ 3 & 2 & 4 \end{vmatrix} = 0\hat{i} - (-10)\hat{j} + (-5)\hat{k} = \boxed{\langle 0, 10, -5 \rangle}$$

By definition of cross product, $\vec{PQ} \times \vec{PR} \perp \vec{PQ} \rightarrow \vec{PQ} \times \vec{PR} \perp \vec{PR}$.

- (ii) Find the area of the triangle PQR . (2 points)

$$\begin{aligned} \text{Area } \Delta PQR &= \frac{1}{2} |\vec{PQ} \times \vec{PR}| = \frac{1}{2} \sqrt{10^2 + 5^2} \\ &= \boxed{\frac{\sqrt{125}}{2}} \quad \left(\text{or } \frac{5\sqrt{5}}{2} \right) \end{aligned}$$

Problem References:

1. MAC2313 L2 HW Assignment Problem #8 and L2 Notes NYTI #2.
2. MAC2313 L3 HW Assignment Problem #6.
3. MAC2313 L4 HW Assignment Problem #7.