MAC 2313 - Period: ____
Quiz 2
January 24, 2019

Name: KEY

Show your work to earn full credit.

1. Find parametric equations for the line of intersection of the planes $x+y+z=3$ and $7x+y+7z=3$. (*2 points*)

Vector in direction of line: $\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 7 & 1 & 7 \end{vmatrix} = \langle 6, 0, -6 \rangle$.

Point on line: Observe that $(0,3,0)$ satisfies both equations.

So vector equation: $\langle x,y,z \rangle = \langle 0,3,0 \rangle + t\langle 6,0,-6 \rangle$ gives parametric eq $\begin{cases} x=6t \\ y=3 \\ z=-6t \end{cases}$

2. Find the equation of the plane that passes through the point $(1,-1,1)$ and contains the line $r(t) = \langle 1,0,-1 \rangle + t\langle 1,2,1 \rangle$. (*4 points*)

Get two vectors in plane then take cross product to find normal vector.

$\vec{v}_1$: Use direction of line $\vec{v}_1 = \langle 1,2,1 \rangle$

$\vec{v}_2$: Use vector from point on line to point $(1,-1,1)$

So $\vec{v}_2 = \langle 1-1, -1-0, 1-(-1) \rangle = \langle 0,-1,2 \rangle$

Then $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & -1 & 2 \end{vmatrix} = \langle 5,-2,-1 \rangle$. So plane has equation

$0 = \vec{n} \cdot \vec{P_0P} = \langle 5,-2,-1 \rangle \cdot \langle x-1, y-(-1), z-1 \rangle \iff$ $5x-2y-z=6$

3. Consider the equation $x^2 - y^2 + z^2 - 4x - 2y - 4z + 6 = 0$.

(*i*) Reduce the given equation to one of the standard forms. (*1.5 points*)

Complete the square:

$x^2-4x+4 \quad -y^2-2y-1 \quad +z^2-4z+4 \quad +6-4+1-4 = 0$

$\iff (x-2)^2 - (y+1)^2 + (z-2)^2 - 1 = 0$

OR $(x-2)^2 - (y+1)^2 + (z-2)^2 = 1$

(*ii*) Classify the surface (i.e., give the name describing the surface). (*0.5 points*)

Hyperboloid of one sheet

Problem References:

1. MAC2313 L5 HW Assignment Problem #11. Answer: $x = 6t, y = 3, z = -6t$.
2. MAC2313 L5 HW Assignment Problem #15. Answer: $5x - 2y - z = 6$.
3. MAC2313 L6 HW Assignment Problem #16 and L6 NYTI #2. Answer: (i) $(x-2)^2 - (y+1)^2 + (z-2)^2 = 1$. (ii) Hyperboloid of one sheet