MAC 2313 - Period: ____ Name: KEY

Quiz 2

January 25, 2018

Show your work to earn full credit.

1. Find the equation of the plane that passes through the point $(1, -1, 8)$ and perpendicular to the line $x = 2t$, $y = 2 - t$, $z = 1 + 5t$. (*3 points*)

The direction of the line is given by $\langle 2, -1, 5\rangle$.

As this line is perpendicular to our desired plane, the vector $\vec{n} = \langle 2,-1,5\rangle$ is normal to our plane. So the equation of the plane is

$$\vec{n}\cdot\langle x-1, y+1, z-8\rangle = 0 \iff \boxed{2x - y + 5z = 43}$$

2. Consider the equation $x^2 + y^2 - 2x - 8y - z + 17 = 0$.

(*i*) Reduce the given equation to one of the standard forms. (*1.5 points*)

$$(x^2 - 2x + 1) + (y^2 - 8y + 16) - z + 17 - 1 - 16 = 0$$

$$\iff (x-1)^2 + (y-4)^2 - z = 0$$

$$\iff \boxed{z = (x-1)^2 + (y-4)^2}$$

(*ii*) Classify the surface. (*0.5 points*)

$\boxed{\text{Elliptic Paraboloid}}$

3. Given $\boldsymbol{u}(t) = \langle \sin(4t), \cos(2t), t\rangle$, and $\boldsymbol{v}(t) = \langle t, \cos(2t), \sin(4t)\rangle$ determine $\frac{d}{dt}[\boldsymbol{u}(t)\cdot\boldsymbol{v}(t)]$. (*3 points*)

From differentiation rules, $\frac{d}{dt}\left[u(t)\cdot v(t)\right] = u'(t)\cdot v(t) + u(t)\cdot v'(t)$.

$u'(t) = \langle 4\cos(4t), -2\sin(2t), 1\rangle$ and $v'(t) = \langle 1, -2\sin(2t), 4\cos(4t)\rangle$.

Substituting $u(t)$, $v(t)$, $u'(t)$, and $v'(t)$ into the formula we get

$$u'(t)\cdot v(t) = 4t\cos(4t) - 2\sin(2t)\cos(2t) + \sin(4t)$$

$$u(t)\cdot v'(t) = \sin(4t) - 2\sin(2t)\cos(2t) + 4t\cos(4t)$$

So

$$\boxed{\frac{d}{dt}\left[u(t)\cdot v(t)\right] = 8t\cos(4t) - 4\sin(2t)\cos(2t) + 2\sin(4t)}$$

Problem References:

1. MAC2313 L5 HW Assignment Problem #7. Answer: $2x - y + 5z = 43$.
2. MAC2313 L6 HW Assignment Problem #12 and L6 NYTI #2. Answer: (i) $z = (x-1)^2 + (y-4)^2$. (ii) Elliptic paraboloid
3. MAC2313 L7 HW Assignment Problem #13. Answer: $2\sin(4t) + 8t\cos(4t) - 4\sin(2t)\cos(2t)$.