MAC 2313 - Period: ___
Quiz 2
January 25, 2018

Name: KEY

Show your work to earn full credit.

1. Find the equation of the plane that passes through the point \((1, -1, 8)\) and perpendicular to the line \(x = 2t, y = 2 - t, z = 1 + 5t\). (3 points)

The direction of the line is given by \(\langle 2, -1, 5 \rangle\).
As this line is perpendicular to our desired plane, the vector \(\vec{n} = \langle 2, -1, 5 \rangle\) is normal to our plane. So the equation of the plane is
\[
\vec{n} \cdot \langle x - 1, y + 1, z - 8 \rangle = 0 \quad \iff \quad 2x - y + 5z = 43
\]

2. Consider the equation \(x^2 + y^2 - 2x - 8y - z + 17 = 0\).
   (i) Reduce the given equation to one of the standard forms. (1.5 points)

\[
(x^2 - 2x + 1) + (y^2 - 8y + 16) - z + 17 - 1 - 16 = 0
\]
\[
\iff \quad (x - 1)^2 + (y - 4)^2 - z = 0
\]
\[
\iff \quad z = (x - 1)^2 + (y - 4)^2
\]

(ii) Classify the surface. (0.5 points)

**Elliptic Paraboloid**

3. Given \(u(t) = \langle \sin(4t), \cos(2t), t \rangle\), and \(v(t) = \langle t, \cos(2t), \sin(4t) \rangle\) determine \(\frac{d}{dt}[u(t) \cdot v(t)]\). (3 points)

From differentiation rules,
\[
\frac{d}{dt} [u(t) \cdot v(t)] = u'(t) \cdot v(t) + u(t) \cdot v'(t).
\]
\[
u'(t) = \langle 4\cos(4t), -2\sin(2t), 1 \rangle \quad \text{and} \quad v'(t) = \langle 1, -2\sin(2t), 4\cos(4t) \rangle.
\]

Substituting \(u(t), v(t), u'(t), \) and \(v'(t)\) into the formula we get
\[
u'(t) \cdot v(t) = 4t\cos(4t) - 2\sin(2t)\cos(2t) + \sin(4t)
\]
\[
u(t) \cdot v'(t) = 8t\sin(4t) - 4\sin(2t)\cos(2t) + 2\sin(4t)
\]

So
\[
\frac{d}{dt} [u(t) \cdot v(t)] = 8t \cos(4t) - 4\sin(2t)\cos(2t) + 2\sin(4t).
\]
Problem References:
1. MAC2313 L5 HW Assignment Problem #7. Answer: \(2x - y + 5z = 43\).
2. MAC2313 L6 HW Assignment Problem #12 and L6 NYTI #2. Answer: (i) \(z = (x - 1)^2 + (y - 4)^2\). (ii) Elliptic paraboloid
3. MAC2313 L7 HW Assignment Problem #13. Answer: \(2\sin(4t) + 8t \cos(4t) - 4 \sin(2t) \cos(2t)\).