MAC 2313 - Period: ____
Quiz 3
February 14, 2019

Name: KEY

Show your work to earn full credit.

1. Let $f(x,y) = \sqrt{xy}$.

(*i*) Find the linear approximation of $f(x,y)$ at the point $(1,4)$. (*3 points*)

$\nabla f(x,y) = \left\langle \frac{y}{2\sqrt{xy}}, \frac{x}{2\sqrt{xy}} \right\rangle \Rightarrow \nabla f(1,4) = \langle 1, \frac{1}{4} \rangle$

$f(1,4) = \sqrt{4} = 2$

$L(x,y) = f(1,4) + \nabla f(1,4) \cdot \langle x-1, y-4 \rangle$

$\Leftrightarrow \boxed{L(x,y) = 2 + (x-1) + \frac{1}{4}(y-4)}$

(*ii*) Using the linearization from part (*i*), estimate $f(0.99, 4.08)$. (*1 point*)

$f(0.99, 4.08) \approx L(0.99, 4.08)$

$= 2 + (0.99-1) + \frac{1}{4}(4.08-4) = 2 - 0.01 + 0.02$

$\approx \boxed{2.01}$

2. Consider the function $f(x,y) = \begin{cases} \dfrac{3x^3y}{x^6+2y^2} & : (x,y) \neq (0,0) \\ 0 & : (x,y) = (0,0) \end{cases}$.

(*i*) Show that $f_x(0,0)$ and $f_y(0,0)$ both exist. (*2 points*) By definition:

$f_x(0,0) = \lim_{h \to 0} \frac{f(h,0) - f(0,0)}{h} = \lim_{h \to 0} \frac{0-0}{h} = 0$

$f_y(0,0) = \lim_{h \to 0} \frac{f(0,h) - f(0,0)}{h} = \lim_{h \to 0} \frac{0-0}{h} = 0.$

(*ii*) Determine if $f(x,y)$ is differentiable at $(0,0)$. You must provide justification to receive full credit. (*2 points*) Observe that

$\underline{x=0}$: $\lim_{y \to 0} \frac{3(0)^3 y}{(0)^6 + 2y^2} = \lim_{y \to 0} \frac{0}{2y^2} = 0$

$\underline{y=x^3}$: $\lim_{x \to 0} \frac{3x^6}{x^6+2x^6} = \lim_{x \to 0} \frac{3x^6}{3x^6} = 1$

So $\lim_{(x,y)\to(0,0)} f(x,y)$ DNE $\Rightarrow$ f not continuous at $(0,0)$ $\Rightarrow$ f not differentiable at $(0,0)$.

Problem References:

1. MAC2313 L13 NYTI #2. Answer: (*i*) $L(x, y) = 2 + (x - 1) + \frac{1}{4}(y - 4)$. (*ii*) 2.01.
2. MAC2313 L13 HW Assignment Problem #14. Answer: (*i*) $f_x(0,0) = f_y(0,0) = 0$. (*ii*) f not continuous at $(0,0)$.