MAC 2313 - Period: ____
Quiz 3
February 15, 2018

Name: KEY

Show your work to earn full credit.

1. Suppose $f(x, y) = \ln(e^2 x^9 y^8)$.

(i) Determine the partial derivatives $f_x(x, y)$ and $f_y(x, y)$. (*2 points*)

$$f_x = \frac{1}{e^2 x^9 y^8} \cdot (9x^8 e^2 y^8) = \boxed{\frac{9}{x}}$$

$$f_y = \frac{1}{e^2 x^9 y^8} \cdot (8y^7 e^2 x^9) = \boxed{\frac{8}{y}}$$

(ii) Using your work from part (i), determine the equation to the tangent plane to the surface $f(x, y)$ at the point $(1, -1, f(1, -1))$. (*2 points*)

Tangent plane: $z - f(1,-1) = f_x(1,-1)(x-1) + f_y(1,-1)(y+1)$

$f(1,-1) = \ln(e^2) = 2$, $f_x(1,-1) = 9$, $f_y(1,-1) = -8$.

$$\boxed{z - 2 = 9(x-1) - 8(y+1)} \quad \text{OR} \quad \boxed{9x - 8y - z = 15}$$

2. Consider the function given by $f(x, y) = \begin{cases} \dfrac{5x^2 y}{x^4 + y^2} & : (x, y) \neq (0, 0) \\ 0 & : (x, y) = (0, 0) \end{cases}$.

(i) Show that $f_x(0, 0)$ and $f_y(0, 0)$ both exist. (*2 points*)

$$f_x(0,0) = \lim_{h \to 0} \frac{1}{h} f(h,0) = \lim_{h \to 0} \frac{1}{h} \cdot \frac{5h^2(0)}{h^4 + (0)^2} = \lim_{h \to 0} \frac{0}{h^5} = 0$$

$$f_y(0,0) = \lim_{h \to 0} \frac{1}{h} f(0,h) = \lim_{h \to 0} \frac{1}{h} \cdot \frac{5(0)^2 h}{(0)^4 + h^2} = \lim_{h \to 0} \frac{0}{h^3} = 0.$$

(normally $\lim_{h \to 0} \frac{f(0,h) - f(0,0)}{h}$ but $f(0,0) = 0$ so I ~~didn't~~ include to save space)

(ii) Show that $f(x, y)$ is not differentiable at $(0, 0)$. (*2 points*)

Observe: Path $y = 0$: $\lim_{(x,y) \to (0,0)} f(x,y) = \lim_{x \to 0} \frac{0}{x^4} = 0$

Path $y = x^2$: $\lim_{(x,y) \to (0,0)} f(x,y) = \lim_{x \to 0} \frac{5x^4}{2x^4} = \frac{5}{2}$.

So $\lim_{(x,y) \to (0,0)} f(x,y)$ DNE $\Rightarrow f$ not continuous at $(0,0) \Rightarrow f$ not differentiable at $(0,0)$.

Problem References:

1. MAC2313 L13 HW Assignment Problem #2 and L13 NYTI #1. Answer: (i) $f_x = \frac{9}{x}$, $f_y = \frac{8}{y}$. (ii) $z = 9x - 8y - 15$.
2. MAC2313 L13 HW Assignment Problem #14. Answer: (i) $f_x(0,0) = 0 = f_y(0,0)$. (ii) f not continuous at $(0,0)$ (need to show)