MAC 2313 - Period: ____ Name: KEY

Quiz 4

February 21, 2019

Show your work to earn full credit.

1. Suppose $f(x,y,z) = \dfrac{x}{y} + yz^2$.

(*i*) Find the gradient of f. (*1.5 points*)

$$\nabla f(x,y,z) = \left\langle \frac{1}{y}, -\frac{x}{y^2} + z^2, 2yz \right\rangle$$

(*ii*) Evaluate the gradient of f at the point $(2,1,-1)$. (*0.5 points*)

$$\nabla f(2,1,-1) = \left\langle \frac{1}{1}, -\frac{2}{1} + (-1)^2, 2(1)(-1) \right\rangle = \langle 1, -1, -2 \rangle$$

(*iii*) Determine the rate of change of f at $(2,1,-1)$ in the direction of $\boldsymbol{v} = \langle 1,2,2 \rangle$. (*2 points*)

$$u = \frac{v}{|v|} = \frac{1}{3}\langle 1,2,2 \rangle \quad \text{since } |v| = \sqrt{1+4+4} = 3.$$

$$\text{So} \quad D_u f = \nabla f(2,1,-1) \cdot u = \langle 1,-1,-2 \rangle \cdot \frac{1}{3}\langle 1,2,2 \rangle$$

$$= \frac{1}{3}(1-2-4) = \boxed{-\frac{5}{3}}$$

2. Find equations of the tangent plane and normal line to the surface $x + y + z = 4e^{xyz}$ at the point $(0,0,4)$. (*4 points*)

Set $F(x,y,z) = x + y + z - 4e^{xyz} = 0$. So

$$\nabla F = \langle 1 - 4yze^{xyz}, 1 - 4xze^{xyz}, 1 - 4xye^{xyz} \rangle.$$

At the point $(0,0,4)$:

$$\nabla F(0,0,4) = \langle 1, 1, 1 \rangle$$

So the tangent plane is given by

$$\nabla F(0,0,4) \cdot \langle x-0, y-0, z-4 \rangle = 0$$

$$\Longleftrightarrow \boxed{x + y + z = 4}$$

The normal line is given by

$$\boxed{\vec{r}(t) = \langle 0,0,4 \rangle + t\langle 1,1,1 \rangle.}$$

Problem References:

1. MAC2313 L15 HW Assignment Problem #9. Answer: (*i*) $\nabla f = \langle \frac{1}{y}, -\frac{x}{y^2} + z^2, 2yz \rangle$. (*ii*) $\langle 1, -1, 2 \rangle$. (*iii*) $-\frac{5}{3}$.
2. MAC2313 L16 HW Assignment Problem #4 and L16 NYTI #1. Answer: Tangent plane: $x + y + z = 4$. Normal line: $(t, t, t + 4)$