MAC 2313 - Period: ____
Quiz 4
February 22, 2018

Name: KEY

Show your work to earn full credit.

1. Suppose $f(x, y, z) = x^2yz - xyz^5$.

(i) Find the gradient of f. (*1.5 points*)

$f_x = 2xyz - yz^5$, $f_y = x^2z - xz^5$, $f_z = x^2y - 5xyz^4$

So $\boxed{\nabla f = \langle 2xyz - yz^5, x^2z - xz^5, x^2y - 5xyz^4 \rangle.}$

(ii) Evaluate the gradient of f at the point $(3, -1, 1)$. (*0.5 points*)

$$\left.\begin{aligned} f_x(3,-1,1) &= 2(3)(-1)(1) - (-1)(1)^5 = -6+1 = -5 \\ f_y(3,-1,1) &= (3)^2(1) - (3)(1)^5 = 9-3 = 6 \\ f_z(3,-1,1) &= (3)^2(-1) - 5(3)(-1)(1)^4 = -9+15 = 6 \end{aligned}\right\} \Rightarrow \boxed{\nabla f(3,-1,1) = \langle -5, 6, 6 \rangle.}$$

(iii) Determine the rate of change of f at $(3, -1, 1)$ in the direction of $v = \langle 0, 4, -3 \rangle$. (*2 points*)

$$u = \frac{v}{|v|} = \frac{1}{\sqrt{0^2+4^2+3^2}}\langle 0,4,-3\rangle = \frac{1}{\sqrt{25}}\langle 0,4,-3\rangle = \langle 0, \tfrac{4}{5}, -\tfrac{3}{5}\rangle.$$

So $$D_uf(3,-1,1) = \langle -5,6,6\rangle \cdot \langle 0, \tfrac{4}{5}, -\tfrac{3}{5}\rangle = 6\left(\tfrac{4}{5} - \tfrac{3}{5}\right) = \boxed{\tfrac{6}{5}}$$

2. Find equations of the tangent plane and normal line to the surface $x + y + z = 3e^{xyz}$ at the point $(0, 0, 3)$. (*4 points*)

Let $F(x,y,z) = x+y+z-3e^{xyz} = 0$. Then

$$\nabla F = \langle 1 - 3yze^{xyz}, 1 - 3xze^{xyz}, 1 - 3xye^{xyz} \rangle$$

$$\nabla F(0,0,3) = \langle 1, 1, 1 \rangle.$$

Tangent plane: $\nabla F(0,0,3) \cdot \langle x-0, y-0, z-3 \rangle = 0$

$$\Leftrightarrow \boxed{x + y + z = 3}$$

Normal line: As $\nabla F(0,0,3) \perp$ to plane, the direction of normal line is $\langle 1,1,1 \rangle$. So

$$\boxed{\langle x,y,z \rangle = \langle 0,0,3 \rangle + t\langle 1,1,1 \rangle}$$

Problem References:

1. MAC2313 L15 HW Assignment Problem #10. Answer: (i) $\nabla f = \langle 2xyz - yz^5, x^2z - xz^5, x^2y - 5xyz^4 \rangle$. (ii) $\langle -5, 6, 6 \rangle$. (iii) $\frac{6}{5}$.
2. MAC2313 L16 HW Assignment Problem #4 and L16 NYTI #1. Answer: Tangent plane: $x + y + z = 3$. Normal line: $(t, t, t + 3)$