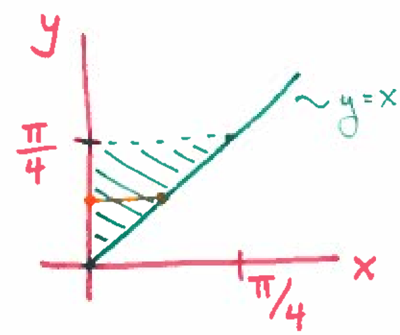
MAC 2313 - Period: ____
Quiz 5
March 21, 2019

Name: KEY

Show your work to earn full credit.

1. Evaluate the integral $\int_0^{\frac{\pi}{4}} \int_x^{\frac{\pi}{4}} \frac{\sin 2y}{y}\, dy\, dx$. (*4 points*)



Changing the order of integration we get

$$\int_0^{\frac{\pi}{4}} \int_0^{y} \frac{\sin 2y}{y}\, dx\, dy$$

$0 \le x \le y$

$0 \le y \le \frac{\pi}{4}$

Inner: $\int_0^y \frac{\sin 2y}{y}\, dx = \frac{\sin 2y}{y}\left[x\right]_0^y = \sin 2y$

Outer: $\int_0^{\pi/4} \sin 2y\, dy = \left[-\frac{1}{2}\cos 2y\right]_0^{\pi/4}$

$= -\frac{1}{2}\left[\cos\frac{\pi}{2} - \cos 0\right] = \boxed{\frac{1}{2}}$

2. Find the volume of the solid above $z = \sqrt{x^2+y^2}$ and below $x^2+y^2+z^2 = 9$. (*4 points*)
① ②

Convert surfaces to polar: ① $z = r$ ② $z = \sqrt{9-r^2}$

Limits of Integration: Solve $r = \sqrt{9-r^2} \Leftrightarrow 2r^2 = 9 \Rightarrow r = \frac{3}{\sqrt{2}}$.

Integral: $\int_0^{2\pi} \int_0^{3/\sqrt{2}} \left(\sqrt{9-r^2} - r\right) r\, dr\, d\theta$

$= \left(\int_0^{2\pi} d\theta\right)\left(\int_0^{3/\sqrt{2}} \left(r(9-r^2)^{1/2} - r^2\right) dr\right)$

$= 2\pi\left[-\frac{1}{3}(9-r^2)^{3/2} - \frac{1}{3}r^3\right]_0^{3/\sqrt{2}}$

$= -\frac{2\pi}{3}\left[\left(9-\frac{9}{2}\right)^{3/2} + \frac{27}{2\sqrt{2}} - (9)^{3/2}\right]$

$= -\frac{2\pi}{3}\left[\frac{27}{2\sqrt{2}} + \frac{27}{2\sqrt{2}} - 27\right]$

$= -\frac{2\pi}{3}\left[\frac{27}{\sqrt{2}} - 27\right] = \boxed{9\pi\left[2-\sqrt{2}\right]}$

Problem References:

1. MAC2313 L21 NYTI #3(a). Answer: $\int_0^{\frac{\pi}{4}} \int_0^{y} \frac{\sin 2y}{y}\, dx\, dy = \frac{1}{2}$.
2. MAC2313 L22 HW Assignment Problem #11 and L22 NYTI #3(b). Answer: $\int_0^{2\pi} \int_0^{\frac{3}{\sqrt{2}}} \left(\sqrt{9 - r^2} - r\right) r\, dr\, d\theta = 9\pi\left(2 - \sqrt{2}\right)$.