MAC 2313 - Period: _____ Name: KEY

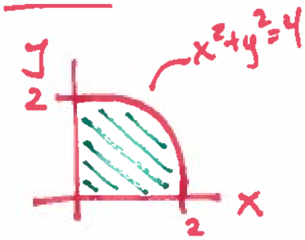
Quiz 6

March 28, 2019

Show your work to earn full credit.

1. Evaluate $\iiint_E z\, dV$ where E is the region between $z = x^2 + y^2$ and $z = 4$ in the first octant. (*4 points*)

Use cylindrical coordinates



Limits: $r^2 = x^2+y^2 \le z \le 4$, $0 \le r \le 2$, $0 \le \theta \le \frac{\pi}{2}$

Integral: $\int_0^{\pi/2} \int_0^2 \int_{r^2}^4 z\, r\, dz\, dr\, d\theta$

Inner: $r\int_{r^2}^4 z\, dz = r\left[\frac{1}{2}z^2\right]_{r^2}^4 = r\left[8 - \frac{r^4}{2}\right] = 8r - \frac{1}{2}r^5$

Middle: $\int_0^2 (8r - \frac{1}{2}r^5)dr = \left[4r^2 - \frac{1}{12}r^6\right]_0^2 = 16 - \frac{16}{3} = \frac{32}{3}$

Outer: $\int_0^{\pi/2} \frac{32}{3}\, d\theta = \frac{\pi}{2}\left(\frac{32}{3}\right) = \boxed{\frac{16\pi}{3}}$

2. Using a triple integral, find the volume of the solid inside of $x^2 + y^2 + z^2 = 9$ and outside of $z = \sqrt{x^2 + y^2}$. (*4 points*)

① ②

Use spherical coordinates

① gives us $\rho = 3$ and ② gives us $\phi = \frac{\pi}{4}$.

Limits: $0 \le \rho \le 3$ (since inside sphere)

$\frac{\pi}{4} \le \phi \le \pi$ (since outside cone)

$0 \le \theta \le 2\pi$ (since not restricted to single octant).

Integral: $\int_0^{2\pi} \int_{\pi/4}^{\pi} \int_0^3 \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

$= \left(\int_0^{2\pi} d\theta\right)\left(\int_{\pi/4}^{\pi} \sin\phi\, d\phi\right)\left(\int_0^3 \rho^2\, d\rho\right)$

$= 2\pi \left[-\cos\phi\right]_{\pi/4}^{\pi} \left[\frac{1}{3}\rho^3\right]_0^3$

$= 2\pi\left[1 + \frac{\sqrt{2}}{2}\right][9] = \boxed{9\pi(2+\sqrt{2})}$

Problem References:

1. MAC2313 L24 HW Assignment Problem #3. Answer: $\int_0^{\pi/2}\int_0^2\int_{r^2}^4 zr\ dz\ dr\ d\theta = \frac{16\pi}{3}$.
2. MAC2313 L25 HW Assignment Problem #9. Answer: $\int_0^{2\pi}\int_{\pi/4}^{\pi}\int_0^3 \rho^2 \sin\phi\ d\rho\ d\phi\ d\theta = 9\pi\left(2+\sqrt{2}\right)$.