1. Evaluate \( \iiint_E z \, dV \) where \( E \) is the region between \( z = x^2 + y^2 \) and \( z = 4 \) in the first octant. (4 points)

**Use cylindrical coordinates**

**Limits:** \( r^2 = x^2 + y^2 \leq z \leq 4 \), \( 0 \leq r \leq 2 \), \( 0 \leq \theta \leq \frac{\pi}{2} \)

**Integral:** \( \int_0^{\frac{\pi}{2}} \int_0^2 \int_r^4 z \, r \, dz \, dr \, d\theta \)

**Inner:** \( \int_r^4 z \, dz = r \left[ \frac{1}{2} z^2 \right]_r^4 = r \left[ 8 - \frac{r^4}{2} \right] = 8r - \frac{1}{2} r^5 \)

**Middle:** \( \int_0^2 (8r - \frac{1}{2} r^5) \, dr = \left[ 4r^2 - \frac{1}{12} r^6 \right]_0^2 = 16 - \frac{16}{3} = \frac{32}{3} \)

**Outer:** \( \int_0^{\frac{\pi}{2}} \frac{32}{3} \, d\theta = \frac{32}{3} \left( \frac{\pi}{2} \right) = \frac{16\pi}{3} \)

2. Using a triple integral, find the volume of the solid inside of \( x^2 + y^2 + z^2 = 9 \) and outside of \( z = \sqrt{x^2 + y^2} \). (4 points)

**Use spherical coordinates**

(1) gives us \( \rho = 3 \) and (2) gives us \( \phi = \frac{\pi}{4} \).

**Limits:** \( 0 \leq \rho \leq 3 \) (since inside sphere), \( \frac{\pi}{4} \leq \phi \leq \pi \) (since outside cone), \( 0 \leq \theta \leq 2\pi \) (since not restricted to single octant).

**Integral:** \( \int_0^{2\pi} \int_{\frac{\pi}{4}}^{\pi} \int_0^{3} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta \)

\( = \left( \int_0^{2\pi} d\theta \right) \left( \int_{\frac{\pi}{4}}^{\pi} \sin \phi \, d\phi \right) \left( \int_0^3 \rho^2 \, d\rho \right) \)

\( = 2\pi \left[ -\cos \phi \right]_{\frac{\pi}{4}}^{\pi} \left[ \frac{1}{3} \rho^3 \right]_0^3 \)

\( = 2\pi \left[ 1 + \frac{\sqrt{2}}{2} \right] [9] = 9\pi \left( 2 + \sqrt{2} \right) \)
Problem References:

1. MAC2313 L24 HW Assignment Problem #3. Answer: \( \int_{0}^{\pi/2} \int_{0}^{2} \int_{r^2}^{3} z r dr dz d\theta = \frac{16\pi}{3} \).

2. MAC2313 L25 HW Assignment Problem #9. Answer: \( \int_{0}^{2\pi} \int_{\pi/4}^{\pi/2} \int_{0}^{3} \rho^2 \sin \phi \ d\rho \ d\phi \ d\theta = 9\pi \left(2 + \sqrt{2}\right) \).