MAC 2313 - Period: ____ Name: Key

Quiz 6

March 29, 2018 **Show your work to earn full credit.**

1. Evaluate $\int\int_E\int e^{-x^2-y^2}$ where E is the region between $x^2+y^2=1$ and $x^2+y^2=4$ with $2 \le z \le 8$ and $y \ge 0$. (*3 points*) Cylindrical coords: $1 \le r \le 2$, $0 \le \theta \le \pi$, $2 \le z \le 8$.

$$\int_0^{\pi}\int_1^2\int_2^8 r e^{-r^2}\, dz\, dr\, d\theta = \left(\int_0^{\pi} d\theta\right)\left(\int_1^2 r e^{-r^2} dr\right)\left(\int_2^8 dz\right)$$

$$= 6\pi\left[-\frac{1}{2}e^{-r^2}\right]_1^2$$

$$= \boxed{3\pi\left(e^{-1}-e^{-4}\right)}$$

Projection onto xy-plane

2. Find the volume of the solid that lies below $z=\sqrt{3x^2+3y^2}$, above the xy-plane, and inside of $x^2+y^2+z^2=9$. (*5 points*)

$z = \sqrt{3x^2+3y^2}$ $\xrightarrow{\text{spherical coords}}$ $\rho\cos\phi = \sqrt{3}\sqrt{\rho^2\sin^2\phi\cos^2\theta + \rho^2\sin^2\phi\sin^2\theta}$

$= \sqrt{3}\,\rho\sin\phi$

$\Leftrightarrow \tan\phi = \frac{1}{\sqrt{3}} \Rightarrow \phi = \frac{\pi}{6}$.

Since below the cone $\tan\phi = \frac{1}{\sqrt{3}}$ but above xy-plane, $\frac{\pi}{6} \le \phi \le \frac{\pi}{2}$.

Inside of $x^2+y^2+z^2=9 \Leftrightarrow$ inside $\rho^2=9 \Rightarrow 0 \le \rho \le 3$.

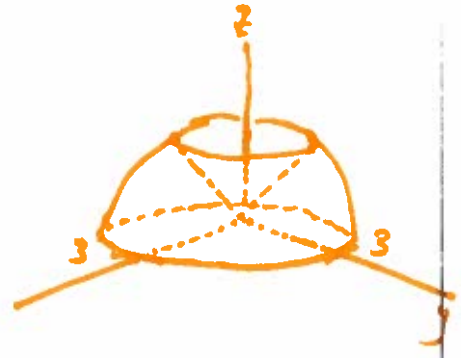
Finally, we have $0 \le \theta \le 2\pi$.

$$\text{Volume} = \int_0^{2\pi}\int_{\pi/6}^{\pi/2}\int_0^3 \rho^2\sin\phi\, d\rho\, d\phi\, d\theta$$

$$= \left(\int_0^{2\pi} d\theta\right)\left(\int_{\pi/6}^{\pi/2}\sin\phi\, d\phi\right)\left(\int_0^3 \rho^2\, d\rho\right)$$

$$= 2\pi\left(\cos\frac{\pi}{6} - \cos\frac{\pi}{2}\right)\left(\frac{1}{3}(3)^3\right)$$

$$= 2\pi\left(\frac{\sqrt{3}}{2}\right)(9) = \boxed{9\pi\sqrt{3}}$$



Problem References:

1. MAC2313 L23 Notes NYTI Problem #2. Answer: $\int_0^{\pi}\int_1^2\int_2^8 re^{-r^2}\,dz\,dr\,d\theta = 3\pi(e^{-1}-e^{-4})$.

2. MAC2313 L24 HW Assignment Problem #13. Answer: $\int_0^{2\pi}\int_{\pi/6}^{\pi/2}\int_0^3 \rho^2\sin\phi\,d\rho\,d\phi\,d\theta = 9\pi\sqrt{3}$.